1. (a) Evaluate \( \lim_{(x,y) \to (1,-1)} \frac{\sin(x^2+y)}{x^2+y} \) or show that it doesn’t exist.
(b) Consider the area function \( A(x) = \int_1^x f(t)dt \), with \( A(2) = 6 \) and \( A(3) = 5 \).
Compute \( \int_3^2 f(t)dt \).
(c) A self-employed software engineer estimates that her annual income over the next 10 years will steadily increase according to the formula \( 70,000e^{0.11t} \), where \( t \) is the time in years. She decides to save 10% of her income in an account paying 6% annual interest, compounded continuously. Treating the savings as a continuous income stream over a 10-year period, find the present value.
(d) Draw the level curves of the graph of \( f(x,y) = 2x^2 + y^2 \) at the heights 0, 1, 2.
(e) Evaluate \( \int_0^1 \frac{\cos(\sqrt{x})}{\sqrt{x}} dx \).
(f) Let \( f(x,y) = \frac{x+y}{x-y} \). Use linear approximation to estimate \( f(2.95, 2.05) \).

2. Evaluate \( \int \frac{x+2}{x(x^2-1)} dx \).

3. Find the area of the region in the first quadrant bounded by \( y = \frac{1}{x} \), \( y = 4x \), and \( y = \frac{1}{2}x \).

4. Find \( k \) such that \( f(x) = \frac{k}{(x+1)^3} \) is a probability density function on the interval \([0, \infty)\), for some random variable \( X \). Then compute the probability that \( 1 \leq X \leq 4 \).

5. Mothballs tend to evaporate at a rate proportional to their surface area. If \( V \) is the volume of a mothball, then its surface area is roughly \( V^{2/3} \). Suppose that the mothball’s volume \( V(t) \) (as a function of times \( t \) in weeks) decreases at a rate that is twice its surface area, and that it initially has a volume of 27 cubic centimeters. Construct and solve an initial value problem for the volume \( V(t) \). Then determine if and when the mothball vanishes.

6. Consider the surface \( z = f(x,y) = 1 + \frac{1}{\sqrt{xy}} \). At the point on the surface above the point \( (x,y) = (4,1) \), what is the direction of steepest descent? Describe this direction with a unit vector in the \( xy \)-plane.

7. By employing \( x \) semi-skilled workers and \( y \) skilled workers, a factory can assemble \( \sqrt{4xy + y^2} \) custom-built computers per hour. The factory pays each semi-skilled worker $8 per hour, and each skilled worker $20 per hour. Determine the maximum number of computers the factory can assemble in an hour for a total labour cost of $720.

8. Find and classify the critical points of \( f(x,y) = 7x^2 - 5xy + y^2 + x - y + 6 \).

9. Given the supply and demand curves
\[ p = D(q) = 8 - q, \quad p = S(q) = \sqrt{q + 1} + 3, \]
find the equilibrium point and the consumer/producer surplus.