1. Determine the type of the following improper integral. Then find out whether it converges or diverges.

\[ \int_{0}^{\infty} \frac{x^2}{x^5 + 1} \, dx. \]

2. Evaluate the integrals

(a) \[ \int \frac{d\theta}{\cos \theta (1 + \sin \theta)} \] and (b) \[ \int_{1}^{e} \sin(\ln x) \, dx. \]

(Answer: (a) \( \frac{1}{4} \ln \left| \frac{1 + \sin \theta}{1 - \sin \theta} \right| - \frac{1}{2(1 + \sin \theta)} + C \), (b) \( \frac{1}{2} (e \sin(1) - e \cos(1) + 1) \))
3. Consider a square plate of edge length $a$, whose density at a point $P$ is equal to $kr$ g/cm$^2$, where $r$ is the distance in centimetres from $P$ to one of the diagonals of the square. Find the mass and center of mass of the plate.

(Answer: $ka^3/3\sqrt{2}$ g, centre of the square.)