6. The area of revolution of \( y = \sqrt{x} \), \((0 \leq x \leq 6)\), about the \( x \)-axis is

\[
S = 2\pi \int_{0}^{6} y \sqrt{1 + \left( \frac{dy}{dx} \right)^2} \, dx
\]

\[
= 2\pi \int_{0}^{6} \sqrt{x} \sqrt{1 + \frac{1}{4x}} \, dx
\]

\[
= 2\pi \int_{0}^{6} \sqrt{x + \frac{1}{4}} \, dx
\]

\[
= \frac{4\pi}{3} \left( x + \frac{1}{4} \right)^{3/2} \bigg|_{0}^{6} = \frac{4\pi}{3} \left[ \frac{125}{8} - \frac{1}{8} \right] = \frac{62\pi}{3} \text{ sq. units.}
\]
Consider the area element which is the thin half-ring shown in the figure. We have
\[ dm = ks \pi s \, ds = k\pi s^2 \, ds. \]
Thus, \( m = \frac{k\pi}{3} a^3. \)

Regard this area element as itself composed of smaller elements at positions given by the angle \( \theta \) as shown. Then
\[
\begin{align*}
\, dM_y &= \left( \int_0^\pi (s \sin \theta) s \, d\theta \right) ks \, ds \\
&= 2ks^3 \, ds, \\
M_y &= 2k \int_0^a s^3 \, ds = \frac{ka^4}{2}.
\end{align*}
\]

Therefore, \( \bar{y} = \frac{ka^4}{2} \cdot \frac{3}{k\pi a^3} = \frac{3a}{2\pi}. \) By symmetry, \( \bar{x} = 0. \) Thus, the centre of mass of the plate is \( \left( 0, \frac{3a}{2\pi} \right). \)
1. a) The \( n \)th bead extends from \( x = (n - 1)\pi \) to \( x = n\pi \), and has volume

\[
V_n = \pi \int_{(n-1)\pi}^{n\pi} e^{-2kx} \sin^2 x \, dx
\]

Let \( x = u + (n - 1)\pi \)

\[
dx = du
\]

\[
= \frac{\pi}{2} \int_{0}^{\pi} e^{-2ku} \left[ 1 - \cos(2u + 2(n - 1)\pi) \right] du
\]

\[
= \frac{\pi}{2} e^{-2k(n-1)\pi} \int_{0}^{\pi} e^{-2ku} (1 - \cos(2u)) \, du
\]

\[
= e^{-2k(n-1)\pi} V_1.
\]

Thus \( \frac{V_{n+1}}{V_n} = \frac{e^{-2kn\pi} V_1}{e^{-2k(n-1)\pi} V_1} = e^{-2k\pi} \), which depends on \( k \) but not \( n \).

b) \( V_{n+1}/V_n = 1/2 \) if \( -2k\pi = \ln(1/2) = -\ln 2 \), that is, if \( k = (\ln 2)/(2\pi) \).

c) Using the result of Example 4 in Section 7.1, we calculate the volume of the first bead:

\[
V_1 = \frac{\pi}{2} \int_{0}^{\pi} e^{-2kx} (1 - \cos(2x)) \, dx
\]

\[
= \frac{\pi}{2} e^{-2k\pi} \int_{0}^{\pi} \left[ 2(\sin(2x) - k\cos(2x)) \right] \frac{1}{4(1 + k^2)} \, dx
\]

\[
= \frac{\pi}{4k(1 + k^2)} (1 - e^{-2k\pi}).
\]

By part (a) and Theorem 1(d) of Section 6.1, the sum of the volumes of the first \( n \) beads is

\[
S_n = \frac{\pi}{4k(1 + k^2)} (1 - e^{-2k\pi})
\]

\[
\times \left[ 1 + e^{-2k\pi} + (e^{-2k\pi})^2 + \cdots + (e^{-2k\pi})^{n-1} \right]
\]

\[
= \frac{\pi}{4k(1 + k^2)} (1 - e^{-2k\pi}) \frac{1 - e^{-2kn\pi}}{1 - e^{-2k\pi}}
\]

\[
= \frac{\pi}{4k(1 + k^2)} (1 - e^{-2kn\pi}).
\]

Thus the total volume of all the beads is

\[
V = \lim_{n \to \infty} S_n = \frac{\pi}{4k(1 + k^2)} \text{ cu. units}.
\]
12. \[ s = \int_{\pi/6}^{\pi/4} \sqrt{1 + \tan^2 x} \, dx \]

\[ = \int_{\pi/6}^{\pi/4} \sec x \, dx = \ln | \sec x + \tan x | \bigg|_{\pi/6}^{\pi/4} \]

\[ = \ln(\sqrt{2} + 1) - \ln \left( \frac{2}{\sqrt{3}} + \frac{1}{\sqrt{3}} \right) \]

\[ = \ln \frac{\sqrt{2} + 1}{\sqrt{3}} \text{ units.} \]
28. The area of the cone obtained by rotating the line
\( y = (h/r)x, \ 0 \leq x \leq r, \) about the y-axis is

\[
S = 2\pi \int_0^r x \sqrt{1 + (h/r)^2} \, dx = 2\pi \frac{\sqrt{r^2 + h^2}}{r} \left[ \frac{r}{2} \right]_0^r
\]

\[
= \pi r \sqrt{r^2 + h^2} \text{ sq. units.}
\]