The ellipses \(3x^2 + 4y^2 = C\) all satisfy the differential equation

\[ 6x + 8y \frac{dy}{dx} = 0, \quad \text{or} \quad \frac{dy}{dx} = -\frac{3x}{4y}. \]

A family of curves that intersect these ellipses at right angles must therefore have slopes given by \(\frac{dy}{dx} = \frac{4y}{3x}\). Thus

\[ 3 \int \frac{dy}{y} = 4 \int \frac{dx}{x} \]

\[ 3 \ln |y| = 4 \ln |x| + \ln |C|. \]

The family is given by \(y^3 = Cx^4\).
8.

Let the disk have centre (and therefore centroid) at \((0, 0)\). Its area is \(9\pi\). Let the hole have centre (and therefore centroid) at \((1, 0)\). Its area is \(\pi\). The remaining part has area \(8\pi\) and centroid at \((\bar{x}, 0)\), where
\[
(9\pi)(0) = (8\pi)\bar{x} + (\pi)(1).
\]
Thus \(\bar{x} = -1/8\). The centroid of the remaining part is \(1/8\) ft from the centre of the disk on the side opposite the hole.
20. \[ y' + (\cos x)y = 2xe^{-\sin x}, \quad y(\pi) = 0 \]
\[
\mu = \int \cos x \, dx = \sin x
\]
\[
\frac{d}{dx}(e^{\sin x}y) = e^{\sin x}(y' + (\cos x)y) = 2x
\]
\[
e^{\sin x}y = \int 2x \, dx = x^2 + C
\]
\[
y(\pi) = 0 \Rightarrow 0 = \pi^2 + C \Rightarrow C = -\pi^2
\]
\[
y = (x^2 - \pi^2)e^{-\sin x}.
\]
17. \[ f_{\mu, \sigma}(x) = \frac{1}{\sigma \sqrt{2\pi}} e^{-(x-\mu)^2/2\sigma^2} \]

mean = \[ \frac{1}{\sigma \sqrt{2\pi}} \int_{-\infty}^{\infty} xe^{-(x-\mu)^2/2\sigma^2} \, dx \quad \text{Let } z = \frac{x - \mu}{\sigma} \quad dz = \frac{1}{\sigma} \, dx \]

= \[ \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} (\mu + \sigma z) e^{-z^2/2} \, dz \]

= \[ \mu \int_{-\infty}^{\infty} e^{-z^2/2} \, dz = \mu \]

variance = \[ E((x - \mu)^2) \]

= \[ \frac{1}{\sigma \sqrt{2\pi}} \int_{-\infty}^{\infty} (x - \mu)^2 e^{-(x-\mu)^2/2\sigma^2} \, dx \]

= \[ \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} \sigma^2 z^2 e^{-z^2/2} \, dz = \sigma \text{Var}(Z) = \sigma \]
9. A layer of water between depths \( y \) and \( y + dy \) has volume \( dV = \pi (a^2 - y^2) \, dy \) and weight \( dF = 9,800 \pi (a^2 - y^2) \, dy \) N. The work done to raise this water to height \( h \) m above the top of the bowl is

\[
dW = (h + y) \, dF = 9,800 \pi (h + y)(a^2 - y^2) \, dy \, \text{N} \cdot \text{m}.
\]

Thus the total work done to pump all the water in the bowl to that height is

\[
W = 9,800 \pi \int_0^a (ha^2 + a^2 y - hy^2 - y^3) \, dy
\]

\[
= 9,800 \pi \left[ \frac{ha^2 y + a^2 y^2}{2} - \frac{hy^3}{3} - \frac{y^4}{4} \right]^a_0
\]

\[
= 9,800 \pi \left[ \frac{2a^3 h}{3} + \frac{a^4}{4} \right]
\]

\[
= 9,800 \pi a^3 \frac{3a + 8h}{12} = 2450 \pi a^3 \left( a + \frac{8h}{3} \right) \, \text{N} \cdot \text{m}.
\]

![Fig. 6-9](image-url)