Math 121 Practice Problem Set 1 for the second midterm
(Based on Chapter 8 and Sections 9.1–9.4)

1. Determine whether the sequence
   \[ a_1 > \sqrt{2}, \quad a_{n+1} = \frac{a_n}{2} + \frac{1}{a_n}, \quad n = 1, 2, 3 \ldots \]
   has a limit. If it does, then find the limit.

2. Do the following series
   \[ \sum_{n=10}^{\infty} \frac{(-1)^{n-1}}{\ln \ln n} \quad \text{and} \quad \sum_{n=1}^{\infty} \frac{n^2 \cos(n\pi)}{1 + n^3} \]
   converge absolutely, converge conditionally or diverge?

3. Find the area of the region inside the curve \( r^2 = 2 \cos 2\theta \) and outside \( r = 1 \).

4. Identify the curve whose polar equation is given by \( r = \sec \theta \tan \theta \).

5. Find the intersections of the pair of curves \( r = \theta \), \( r = \theta + \pi \).

6. Find the volume of the solid obtained by rotating about the \( x \)-axis
   the region bounded by that axis and one arch of the cycloid \( x = at - a \sin t \), \( y = a - a \cos t \).

7. Does the alternating series test continue to hold if the assumption
   \[ |a_{n+1}| \leq |a_n| \quad \text{for all} \ n \geq N \]
   is dropped? Prove this statement if it is true, or give a counterexample.