Homework 1 - Math 440/508, Fall 2011

Due on Monday September 19

1. Assume a function \( f \) is analytic in a domain \( D \) and satisfies at least one of the following two conditions:
   i. \( \text{Arg} f(z) = \alpha \forall z \in D \), or
   ii. \( v(z) = u(z)^2 \forall z \in D \).
   Prove that \( f \) is constant on \( D \).

2. Are the following sets connected or disconnected? Give reasons for your answer.
   i. \( F = \{ z : \text{Im}(z)/\text{Re}(z) \in \mathbb{Q} \} \)
   ii. \( A = \mathbb{C} \{ z : \text{Re}(z), \text{Im}(z) \in \mathbb{Q} \} \)

3. Find the entire function \( f(z) = u(z) + iv(z) \) such that \( f(0) = i \) and \( u(z) = 2x^3y - 2xy^3 + x^2 - y^2 \).

4. From Chapter 1, Complex Analysis - Stein & Shakarchi: 7, 9, 16(e), 22, 25