1. An ellipsoid is created by rotating the ellipse $4x^2 + y^2 = 16$ about the $x$-axis. Find an equation for the ellipsoid.

2. Find the unit tangent, normal and binormal vectors of the curve

$$r(t) = \langle \cos t, \sin t, \ln \cos t \rangle$$

at the point $(1, 0, 0)$.

3. Find equations of the normal plane and osculating plane of the curve

$$x = t, \quad y = t^2, \quad z = t^3 \quad \text{at} \quad (1, 1, 1).$$

4. Find the best linear approximation of the function $f(x, y) = \sqrt{y + \cos^2 x}$ at $(0, 0)$.

5. If $x, y, z$ satisfy the relation

$$xyz = \cos(x + y + z)$$

find $\partial z/\partial x$ and $\partial z/\partial y$.

6. Find the dimensions of a rectangular box of largest volume such that the sum of the lengths of its twelve edges is a constant $c$.

7. Evaluate the integral

$$\int_{-2}^{2} \int_{0}^{\sqrt{4-y^2}} \int_{-\sqrt{4-x^2-y^2}}^{\sqrt{4-x^2-y^2}} y^2 \sqrt{x^2 + y^2 + z^2} \, dz \, dx \, dy.$$ 

8. Give five other iterated integrals that are equal to

$$\int_{0}^{2} \int_{0}^{y^3} \int_{0}^{y^2} f(x, y, z) \, dz \, dx \, dy.$$ 

9. Show that there is no vector field $\mathbf{G}$ such that

$$\text{curl } \mathbf{G} = 2x\mathbf{i} + 3yz\mathbf{j} - xz^2\mathbf{k}.$$ 

10. Evaluate the surface integral

$$\iint_{S} (x^2z + y^2z) \, dS$$

where $S$ is part of the plane $z = 4 + x + y$ that lies inside the cylinder $x^2 + y^2 = 4$.

11. Use Stokes’ theorem to evaluate $\iint_{S} \text{curl } \mathbf{F} \cdot d\mathbf{S}$ where $\mathbf{F}(x, y, z) = x^2yz\mathbf{i} + yz^2\mathbf{j} + z^3e^{-y}\mathbf{k}$, $S$ is the part of the sphere $x^2 + y^2 + z^2 = 5$ that lies above the plane $z = 1$ and $S$ is oriented upward.

12. Use the divergence theorem to calculate the surface integral $\iint_{S} \mathbf{F} \cdot d\mathbf{S}$, where $\mathbf{F}(x, y, z) = x^3\mathbf{i} + y^3\mathbf{j} + z^3\mathbf{k}$ and $S$ is the surface of the solid bounded by the cylinder $x^2 + y^2 = 1$ and the planes $z = 0$ and $z = 2$. 