

Math 217 Assignment 4

Due Friday October 9

■ Problems from the text (do NOT turn in these problems):

- Section 15.1: 11-20, 74.
- Section 15.2: 34–38, 10-22, 44
- Section 15.3: 30-38, 73-84, 89–94.
- Section 15.4: 2-6, 12-16, 32-34, 39–42.
- Section 15.5: 20-34, 40-44, 50-56.

■ Problems to turn in:

1. Draw a contour map for the function $f(x, y) = y \sec x$ showing several level curves.
2. Determine whether the function

$$f(x, y) = \begin{cases} \frac{xy}{x^2+xy+y^2} & \text{if } (x, y) \neq (0, 0), \\ 0 & \text{if } (x, y) = (0, 0) \end{cases}$$

is continuous at every point.

3. Does the limit

$$\lim_{(x,y,z) \rightarrow (0,0,0)} \frac{yz}{x^2 + 4y^2 + 9z^2}.$$

exist? If so, find it.

4. Repeat the same question above for

$$\lim_{(x,y) \rightarrow (0,0)} \frac{\sin(x^2 + y^2)}{x^2 + y^2}.$$

5. The temperature at a point (x, y) on a flat metal plate is given by

$$T(x, y) = \frac{60}{1 + x^2 + y^2},$$

where T is measured in $^{\circ}\text{C}$ and x, y in meters. Find the rate of change of temperature with respect to distance at the point $(2, 1)$ in the x direction and the y -direction.

6. Find an equation of the tangent plane to the surface $z = e^{x^2-y^2}$ at the point $(1, -1, 1)$.
7. Suppose you need to know an equation of the tangent plane to a surface S at the point $P(2, 1, 3)$. You don't have an equation for S but you know that the curves

$$\mathbf{r}_1(t) = \langle 2 + 3t, 1 - t^2, 3 - 4t + t^2 \rangle,$$

$$\mathbf{r}_2(u) = \langle 1 + u^2, 2u^3 - 1, 2u + 1 \rangle$$

both lie in S . Find an equation for the tangent plane at P .

8. Four positive numbers, each less than 50, are rounded to the first decimal place and then multiplied together. Use differentials to estimate the maximum possible error in the computed product that may result from the rounding.

9. Calculate $\frac{\partial M}{\partial u}$, $\frac{\partial M}{\partial v}$ when $u = 3$, $v = -1$, and

$$M = xe^{y-z^2}, \quad x = 2uv, \quad y = u - v, \quad z = u + v.$$

10. If a sound with frequency f_s is produced by a source traveling along a line with speed v_s and an observer is traveling with speed v_o along the same line from the opposite direction toward the source, then the frequency of the sound heard by the observer is

$$f_o = \left(\frac{c + v_o}{c - v_s} \right) f_s,$$

where c is the speed of sound, about 332 m/s. This is known as the *Doppler effect*.

Suppose that at a particular moment you are in a train traveling at 34 m/s and accelerating at 1.2 m/s^2 . A train is approaching you from the opposite direction on the other track at 40 m/s, accelerating at 1.4 m/s^2 , and sounds its whistle, which has a frequency of 460 Hz. At that instant, what is the perceived frequency that you hear and how fast is it changing?