12. Let C be the curve of intersection. The projection of C onto the xy-plane is the ellipse $4x^2 + y^2 = 4$ or $x^2 + y^2/4 = 1$, $z = 0$. Then we can write $x = \cos t$, $y = 2\sin t$, $0 \le t \le 2\pi$. Since C also lies on the plane $x + y + z = 2$, we have $z = 2 - x - y = 2 - \cos t - 2\sin t$. Then parametric equations for C are $x = \cos t$, $y = 2\sin t$, $z = 2 - \cos t - 2\sin t$, $0 \le t \le 2\pi$, and the corresponding vector equation is $\mathbf{r}(t) = \langle \cos t, 2\sin t, 2 - \cos t - 2\sin t \rangle$. Differentiating gives

$\mathbf{r}'(t) = \langle -\sin t, 2\cos t, \sin t - 2\cos t \rangle \quad \Rightarrow$

$|\mathbf{r}'(t)| = \sqrt{(-\sin t)^2 + (2\cos t)^2 + (\sin t - 2\cos t)^2} = \sqrt{2\sin^2 t + 8\cos^2 t - 4\sin t\ \cos t}$. The length of C is

$L = \int_0^{2\pi} |\mathbf{r}'(t)|\, dt = \int_0^{2\pi} \sqrt{2\sin^2 t + 8\cos^2 t - 4\sin t\ \cos t}\, dt \approx 13.5191.$

24. $\mathbf{r}(t) = \langle e^t \cos t, e^t \sin t, t \rangle \quad \Rightarrow \quad \mathbf{r}'(t) = \langle e^t \cos t - e^t \sin t, e^t \cos t + e^t \sin t, 1 \rangle$. The point $(1, 0, 0)$ corresponds to $t = 0$, and $\mathbf{r}'(0) = \langle 1, 1, 1 \rangle \quad \Rightarrow \quad |\mathbf{r}'(0)| = \sqrt{1^2 + 1^2 + 1^2} = \sqrt{3}$.

$\mathbf{r}''(t) = \langle e^t \cos t - e^t \sin t - e^t \cos t - e^t \sin t, e^t \cos t - e^t \sin t + e^t \cos t + e^t \sin t, 0 \rangle = \langle -2e^t \sin t, 2e^t \cos t, 0 \rangle \quad \Rightarrow$

$\mathbf{r}''(0) = \langle 0, 2, 0 \rangle$. $\mathbf{r}'(0) \times \mathbf{r}''(0) = \langle -2, 0, 2 \rangle$. $|\mathbf{r}'(0) \times \mathbf{r}''(0)| = \sqrt{(-2)^2 + 0^2 + 2^2} = \sqrt{8} = 2\sqrt{2}$.

Then $\kappa(0) = \dfrac{|\mathbf{r}'(0) \times \mathbf{r}''(0)|}{|\mathbf{r}'(0)|^3} = \dfrac{2\sqrt{2}}{(\sqrt{3})^3} = \dfrac{2\sqrt{2}}{3\sqrt{3}}$ or $\dfrac{2\sqrt{6}}{9}$.

28. Here $v_0 = 115$ ft/s, the angle of elevation is $\alpha = 50°$, and if we place the origin at home plate, then $\mathbf{r}(0) = 3\,\mathbf{j}$. As in Example 5, we have $\mathbf{r}(t) = -\frac{1}{2}gt^2\,\mathbf{j} + t\,\mathbf{v}_0 + \mathbf{D}$ where $\mathbf{D} = \mathbf{r}(0) = 3\,\mathbf{j}$ and $\mathbf{v}_0 = v_0 \cos\alpha\,\mathbf{i} + v_0 \sin\alpha\,\mathbf{j}$, so $\mathbf{r}(t) = (v_0 \cos\alpha)t\,\mathbf{i} + \left[(v_0 \sin\alpha)t - \frac{1}{2}gt^2 + 3\right]\mathbf{j}$. Thus, parametric equations for the trajectory of the ball are $x = (v_0 \cos\alpha)t$, $y = (v_0 \sin\alpha)t - \frac{1}{2}gt^2 + 3$. The ball reaches the fence when $x = 400 \quad \Rightarrow$

$(v_0 \cos\alpha)t = 400 \quad \Rightarrow \quad t = \dfrac{400}{v_0 \cos\alpha} = \dfrac{400}{115\cos 50°} \approx 5.41$ s. At this time, the height of the ball is

$y = (v_0 \sin\alpha)t - \frac{1}{2}gt^2 + 3 \approx (115\sin 50°)(5.41) - \frac{1}{2}(32)(5.41)^2 + 3 \approx 11.2$ ft. Since the fence is 10 ft high, the ball clears the fence.

32. As in Exercise 31(b), let α be the angle north of east that the boat heads, so the velocity of the boat in still water is given by $5(\cos\alpha)\,\mathbf{i} + 5(\sin\alpha)\,\mathbf{j}$. At t seconds, the boat is $5(\cos\alpha)t$ meters from the west bank, at which point the velocity of the water is $3\sin(\pi x/40)\,\mathbf{j} = 3\sin[\pi\cdot 5(\cos\alpha)t/40]\,\mathbf{j} = 3\sin\left(\frac{\pi}{8}t\cos\alpha\right)\mathbf{j}$. The resultant velocity of the boat then is given by $\mathbf{v}(t) = 5(\cos\alpha)\,\mathbf{i} + \left[5\sin\alpha + 3\sin\left(\frac{\pi}{8}t\cos\alpha\right)\right]\mathbf{j}$. Integrating,

$$\mathbf{r}(t) = (5t\cos\alpha)\,\mathbf{i} + \left[5t\sin\alpha - \frac{24}{\pi\cos\alpha}\cos\left(\tfrac{\pi}{8}t\cos\alpha\right)\right]\mathbf{j} + \mathbf{C}.$$

If we place the origin at A then $\mathbf{r}(0) = \mathbf{0} \;\Rightarrow\; -\dfrac{24}{\pi\cos\alpha}\mathbf{j} + \mathbf{C} = \mathbf{0} \;\Rightarrow\; \mathbf{C} = \dfrac{24}{\pi\cos\alpha}\mathbf{j}$ and

$$\mathbf{r}(t) = (5t\cos\alpha)\,\mathbf{i} + \left[5t\sin\alpha - \frac{24}{\pi\cos\alpha}\cos\left(\tfrac{\pi}{8}t\cos\alpha\right) + \frac{24}{\pi\cos\alpha}\right]\mathbf{j}.$$

The boat will reach the east bank when $5t\cos\alpha = 40 \;\Rightarrow\; t = \dfrac{8}{\cos\alpha}$.

In order to land at point $B(40,0)$ we need $5t\sin\alpha - \dfrac{24}{\pi\cos\alpha}\cos\left(\frac{\pi}{8}t\cos\alpha\right) + \dfrac{24}{\pi\cos\alpha} = 0 \;\Rightarrow$

$$5\left(\frac{8}{\cos\alpha}\right)\sin\alpha - \frac{24}{\pi\cos\alpha}\cos\left[\frac{\pi}{8}\left(\frac{8}{\cos\alpha}\right)\cos\alpha\right] + \frac{24}{\pi\cos\alpha} = 0 \;\Rightarrow\; \frac{1}{\cos\alpha}\left(40\sin\alpha - \frac{24}{\pi}\cos\pi + \frac{24}{\pi}\right) = 0 \;\Rightarrow$$

$40\sin\alpha + \dfrac{48}{\pi} = 0 \;\Rightarrow\; \sin\alpha = -\dfrac{6}{5\pi}$. Thus $\alpha = \sin^{-1}\left(-\dfrac{6}{5\pi}\right) \approx -22.5^\circ$, so the boat should head 22.5° south of east.

38. $\mathbf{r}(t) = t\,\mathbf{i} + \cos^2 t\,\mathbf{j} + \sin^2 t\,\mathbf{k} \;\Rightarrow\; \mathbf{r}'(t) = \mathbf{i} - 2\cos t\sin t\,\mathbf{j} + 2\sin t\cos t\,\mathbf{k} = \mathbf{i} - \sin 2t\,\mathbf{j} + \sin 2t\,\mathbf{k}$,

$|\mathbf{r}'(t)| = \sqrt{1 + 2\sin^2 2t}$, $\mathbf{r}''(t) = 2(\sin^2 t - \cos^2 t)\,\mathbf{j} + 2(\cos^2 t - \sin^2 t)\,\mathbf{k} = -2\cos 2t\,\mathbf{j} + 2\cos 2t\,\mathbf{k}$. So

$$a_T = \frac{2\sin 2t\cos 2t + 2\sin 2t\cos 2t}{\sqrt{1 + 2\sin^2 2t}} = \frac{4\sin 2t\cos 2t}{\sqrt{1 + 2\sin^2 2t}} \text{ and } a_N = \frac{|-2\cos 2t\,\mathbf{j} - 2\cos 2t\,\mathbf{k}|}{\sqrt{1 + 2\sin^2 2t}} = \frac{2\sqrt{2}\,|\cos 2t|}{\sqrt{1 + 2\sin^2 t}}.$$

6. (a) C intersects the xz-plane where $y = 0 \;\Rightarrow\; 2t - 1 = 0 \;\Rightarrow\; t = \frac{1}{2}$, so the point is $\left(2 - \left(\frac{1}{2}\right)^3, 0, \ln\frac{1}{2}\right) = \left(\frac{15}{8}, 0, -\ln 2\right)$.

(b) The curve is given by $\mathbf{r}(t) = \langle 2 - t^3, 2t - 1, \ln t\rangle$, so $\mathbf{r}'(t) = \langle -3t^2, 2, 1/t\rangle$. The point $(1,1,0)$ corresponds to $t = 1$, so the tangent vector there is $\mathbf{r}'(1) = \langle -3, 2, 1\rangle$. Then the tangent line has direction vector $\langle -3, 2, 1\rangle$ and includes the point $(1,1,0)$, so parametric equations are $x = 1 - 3t$, $y = 1 + 2t$, $z = t$.

(c) The normal plane has normal vector $\mathbf{r}'(1) = \langle -3, 2, 1\rangle$ and equation $-3(x-1) + 2(y-1) + z = 0$ or $3x - 2y - z = 1$.

10. The parametric value corresponding to the point $(1,0,1)$ is $t = 0$.

$\mathbf{r}'(t) = e^t\,\mathbf{i} + e^t(\cos t + \sin t)\,\mathbf{j} + e^t(\cos t - \sin t)\,\mathbf{k} \;\Rightarrow\; |\mathbf{r}'(t)| = e^t\sqrt{1 + (\cos t + \sin t)^2 + (\cos t - \sin t)^2} = \sqrt{3}\,e^t$

and $s(t) = \int_0^t e^u\sqrt{3}\,du = \sqrt{3}(e^t - 1) \;\Rightarrow\; t = \ln\left(1 + \frac{1}{\sqrt{3}}s\right)$.

Therefore, $\mathbf{r}(t(s)) = \left(1 + \frac{1}{\sqrt{3}}s\right)\mathbf{i} + \left(1 + \frac{1}{\sqrt{3}}s\right)\sin\ln\left(1 + \frac{1}{\sqrt{3}}s\right)\mathbf{j} + \left(1 + \frac{1}{\sqrt{3}}s\right)\cos\ln\left(1 + \frac{1}{\sqrt{3}}s\right)\mathbf{k}$.