38. $\mathbf{n}_1 = \langle 1, 0, -1 \rangle$ and $\mathbf{n}_2 = \langle 0, 1, 2 \rangle$. Setting $z = 0$, it is easy to see that $(1, 3, 0)$ is a point on the line of intersection of $x - z = 1$ and $y + 2z = 3$. The direction of this line is $\mathbf{v}_1 = \mathbf{n}_1 \times \mathbf{n}_2 = \langle 1, -2, 1 \rangle$. A second vector parallel to the desired plane is $\mathbf{v}_2 = \langle 1, 1, -2 \rangle$, since it is perpendicular to $x + y - 2z = 1$. Therefore, a normal of the plane in question is $\mathbf{n} = \mathbf{v}_1 \times \mathbf{v}_2 = \langle 4 - 1, 1 + 2, 1 + 2 \rangle = \langle 3, 3, 3 \rangle$, or we can use $\langle 1, 1, 1 \rangle$. Taking $(x_0, y_0, z_0) = (1, 3, 0)$, the equation we are looking for is $(x - 1) + (y - 3) + z = 0 \quad \Leftrightarrow \quad x + y + z = 4$.

76. First notice that if two lines are skew, they can be viewed as lying in two parallel planes and so the distance between the skew lines would be the same as the distance between these parallel planes. The common normal vector to the planes must be perpendicular to both $\mathbf{v}_1 = \langle 1, 6, 2 \rangle$ and $\mathbf{v}_2 = \langle 2, 15, 6 \rangle$, the direction vectors of the two lines respectively. Thus set $\mathbf{n} = \mathbf{v}_1 \times \mathbf{v}_2 = \langle 36 - 30, 4 - 6, 15 - 12 \rangle = \langle 6, -2, 3 \rangle$. Setting $t = 0$ and $s = 0$ gives the points $(1, 1, 0)$ and $(1, 5, -2)$. So in the notation of Equation 8, $6 - 2 + 0 + d_1 = 0 \quad \Rightarrow \quad d_1 = -4$ and $6 - 10 - 6 + d_2 = 0 \quad \Rightarrow \quad d_2 = 10$.

Then by Exercise 73, the distance between the two skew lines is given by $D = \dfrac{|-4 - 10|}{\sqrt{36 + 4 + 9}} = \dfrac{14}{7} = 2$.

Alternate solution (without reference to planes): We already know that the direction vectors of the two lines are $\mathbf{v}_1 = \langle 1, 6, 2 \rangle$ and $\mathbf{v}_2 = \langle 2, 15, 6 \rangle$. Then $\mathbf{n} = \mathbf{v}_1 \times \mathbf{v}_2 = \langle 6, -2, 3 \rangle$ is perpendicular to both lines. Pick any point on each of the lines, say $(1, 1, 0)$ and $(1, 5, -2)$, and form the vector $\mathbf{b} = \langle 0, 4, -2 \rangle$ connecting the two points. Then the distance between the two skew lines is the absolute value of the scalar projection of $\mathbf{b}$ along $\mathbf{n}$, that is,

$$D = \frac{|\mathbf{n} \cdot \mathbf{b}|}{|\mathbf{n}|} = \frac{1}{\sqrt{36 + 4 + 9}} |0 - 8 - 6| = \frac{14}{7} = 2.$$

46. Let $P = (x, y, z)$ be an arbitrary point whose distance from the x-axis is twice its distance from the yz-plane. The distance from P to the x-axis is $\sqrt{(x - x)^2 + y^2 + z^2} = \sqrt{y^2 + z^2}$ and the distance from P to the yz-plane $(x = 0)$ is $|x|/1 = |x|$. Thus $\sqrt{y^2 + z^2} = 2|x| \quad \Leftrightarrow \quad y^2 + z^2 = 4x^2 \quad \Leftrightarrow \quad x^2 = (y^2/2^2) + (z^2/2^2)$. So the surface is a right circular cone with vertex the origin and axis the x-axis.

50. Any point on the curve of intersection must satisfy both $2x^2 + 4y^2 - 2z^2 + 6x = 2$ and $2x^2 + 4y^2 - 2z^2 - 5y = 0$. Subtracting, we get $6x + 5y = 2$, which is linear and therefore the equation of a plane. Thus the curve of intersection lies in this plane.

42. The particles collide provided $\mathbf{r}_1(t) = \mathbf{r}_2(t) \quad \Leftrightarrow \quad \langle t, t^2, t^3 \rangle = \langle 1+2t, 1+6t, 1+14t \rangle$. Equating components gives $t = 1 + 2t$, $t^2 = 1 + 6t$, and $t^3 = 1 + 14t$. The first equation gives $t = -1$, but this does not satisfy the other equations, so the particles do not collide. For the paths to intersect, we need to find a value for t and a value for s where $\mathbf{r}_1(t) = \mathbf{r}_2(s) \quad \Leftrightarrow$ $\langle t, t^2, t^3 \rangle = \langle 1+2s, 1+6s, 1+14s \rangle$. Equating components, $t = 1 + 2s$, $t^2 = 1 + 6s$, and $t^3 = 1 + 14s$. Substituting the first equation into the second gives $(1+2s)^2 = 1 + 6s \quad \Rightarrow \quad 4s^2 - 2s = 0 \quad \Rightarrow \quad 2s(2s-1) = 0 \quad \Rightarrow \quad s = 0$ or $s = \frac{1}{2}$. From the first equation, $s = 0 \quad \Rightarrow \quad t = 1$ and $s = \frac{1}{2} \quad \Rightarrow \quad t = 2$. Checking, we see that both pairs of values satisfy the third equation. Thus the paths intersect twice, at the point $(1, 1, 1)$ when $s = 0$ and $t = 1$, and at $(2, 4, 8)$ when $s = \frac{1}{2}$ and $t = 2$.

26. $\mathbf{r}(t) = \langle \ln t, 2\sqrt{t}, t^2 \rangle$, $\mathbf{r}'(t) = \langle 1/t, 1/\sqrt{t}, 2t \rangle$. At $(0, 2, 1)$, $t = 1$ and $\mathbf{r}'(1) = \langle 1, 1, 2 \rangle$. Thus, parametric equations of the tangent line are $x = t$, $y = 2 + t$, $z = 1 + 2t$.

32. To find the point of intersection, we must find the values of t and s which satisfy the following three equations simultaneously: $t = 3 - s$, $1 - t = s - 2$, $3 + t^2 = s^2$. Solving the last two equations gives $t = 1$, $s = 2$ (check these in the first equation). Thus the point of intersection is $(1, 0, 4)$. To find the angle θ of intersection, we proceed as in Exercise 31. The tangent vectors to the respective curves at $(1, 0, 4)$ are $\mathbf{r}_1'(1) = \langle 1, -1, 2 \rangle$ and $\mathbf{r}_2'(2) = \langle -1, 1, 4 \rangle$. So

$\cos\theta = \frac{1}{\sqrt{6}\sqrt{18}}(-1 - 1 + 8) = \frac{6}{6\sqrt{3}} = \frac{1}{\sqrt{3}}$ and $\theta = \cos^{-1}\left(\frac{1}{\sqrt{3}}\right) \approx 55°$.

Note: In Exercise 31, the curves intersect when the value of both parameters is zero. However, as seen in this exercise, it is not necessary for the parameters to be of equal value at the point of intersection.

34.
$$\int_0^1 \left(\frac{4}{1+t^2}\,\mathbf{j} + \frac{2t}{1+t^2}\,\mathbf{k}\right) dt = \left[4\tan^{-1} t\,\mathbf{j} + \ln(1+t^2)\,\mathbf{k}\right]_0^1 = \left[4\tan^{-1} 1\,\mathbf{j} + \ln 2\,\mathbf{k}\right] - \left[4\tan^{-1} 0\,\mathbf{j} + \ln 1\,\mathbf{k}\right]$$
$$= 4\left(\tfrac{\pi}{4}\right)\mathbf{j} + \ln 2\,\mathbf{k} - 0\,\mathbf{j} - 0\,\mathbf{k} = \pi\,\mathbf{j} + \ln 2\,\mathbf{k}$$