Math 217 Formula Sheet

1. **Volume of a parallelepiped determined by the vectors a, b and c**:
$$V = |\mathbf{a} \cdot (\mathbf{b} \times \mathbf{c})|.$$

2. **Distance D from a point $P(x_1, y_1, z_1)$ to the plane $ax + by + cz + d = 0$**:
$$D = \frac{|ax_1 + by_1 + cz_1 + d|}{\sqrt{a^2 + b^2 + c^2}}.$$

3. **Curvature of a curve**:
$$\kappa = \left|\frac{d\mathbf{T}}{ds}\right| = \frac{|\mathbf{T}'(t)|}{|\mathbf{r}'(t)|} = \frac{|\mathbf{r}'(t) \times \mathbf{r}''(t)|}{|\mathbf{r}'(t)|^3}.$$

4. **Tangential and normal components of acceleration**:
$$\mathbf{a} = v'\mathbf{T} + \kappa v^2 \mathbf{N}, \qquad v = |\mathbf{r}'(t)|.$$

5. **Second derivatives test for max/min**: Suppose the second partial derivatives of f are continuous on a disk with center (a, b), and suppose that $\nabla f(a, b) = \mathbf{0}$. Let
$$D = f_{xx}(a, b) f_{yy}(a, b) - [f_{xy}(a, b)]^2.$$
   (a) If $D > 0$ and $f_{xx}(a, b) > 0$, then $f(a, b)$ is a local min.
   (b) If $D > 0$ and $f_{xx}(a, b) < 0$, then $f(a, b)$ is a local max.
   (c) If $D < 0$ then $f(a, b)$ is not a local max or min.

6. **Expectation of a random variable**: If X and Y are random variables with joint density f, the expected values of X and Y are
$$\mu_1 = \iint_{\mathbb{R}^2} x f(x, y)\, dA,$$
$$\mu_2 = \iint_{\mathbb{R}^2} y f(x, y)\, dA.$$

7. **Center of mass**: The coordinates $(\bar{x}, \bar{y})$ of the center of mass of a lamina occupying the region D and having density function $\rho(x, y)$ are
$$\bar{x} = \frac{1}{m} \iint_D x\rho(x, y)\, dA, \qquad \bar{y} = \frac{1}{m} \iint_D y\rho(x, y)\, dA$$
$$m = \iint_D \rho(x, y)\, dA.$$

8. **Fundamental Theorem of Line Integrals**: If f is a scalar function with continuous first partial derivarives, and $C = \{\mathbf{r}(t) : a \leq t \leq b\}$ is a smooth curve, then
$$\int_C \nabla f \cdot d\mathbf{r} = f(\mathbf{r}(b)) - f(\mathbf{r}(a)).$$

9. **Green's Theorem**: Let C be a positively oriented, simple, closed curve in the plane and D the region bounded by C. If P and Q have continuous partial derivatives on an open region that contains D, then
$$\oint_C P dx + Q\, dy = \iint_D \left(\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y} \right)\, dA.$$

10. **Stokes' Theorem**: Let S be an oriented piecewise smooth surface that is bounded by a simple closed piecewise smooth boundary curve C with positive orientation. Let $\mathbf{F}$ be a vector field whose components have continuous partial derivatives on an open region in $\mathbb{R}^3$ that contains S. Then
$$\int_C \mathbf{F} \cdot d\mathbf{r} = \iint_S \operatorname{curl} \mathbf{F} \cdot d\mathbf{S}.$$

11. **Divergence Theorem**: Let E be a simple solid region and let S be the boundary surface of E, given with positive (outward) orientation. Let $\mathbf{F}$ be a vector field whose component functions have continuous partial derivatives on an open region that contains E. Then
$$\iint_S \mathbf{F} \cdot d\mathbf{S} = \iiint_E \operatorname{div} \mathbf{F} dV.$$