Problems from the text (do NOT turn in these problems):
(15.1) 13-18, 23-27, 30-42, 55-60. (15.2) 5-12. (15.3) 5-10, 15-20, 51-56, 70, 74-75. (15.4) 1-6, 11-20, 40, 41, 42. (15.5) 1-12, 21-34, 40.

Problems to turn in:

1. (a) Draw a contour diagram for the function \( f(x, y) = \sqrt{(x - 1)^2 + (y - 2)^2} \). Indicate the contours \( f(x, y) = 1, 2, 3 \) and \( 4 \).
(b) Calculate \( \nabla f(2, 3) \) and indicate this vector on your diagram.
(c) Consider \( z = f(x, y) \). Find the equation of the tangent plane to \( f(x, y) \) at the point \( (2, 3) \).

2. A function \( z = f(x, y) \) is called harmonic if it satisfies this equation:
\[
\frac{\partial^2 z}{\partial x^2} + \frac{\partial^2 z}{\partial y^2} = 0.
\]
This is called Laplace’s Equation. Determine whether or not the following functions are harmonic:
(a) \( z = \sqrt{x^2 + y^2} \)
(b) \( e^{-x} \sin y \)
(c) \( 3x^2y - y^3 \)

3. In each case, give an example of an appropriate function or show that no such function exists.
(a) A function \( f(x, y) \) with continuous second order partial derivatives and which satisfies \( \frac{\partial f}{\partial x} = 6xy^2 \) and \( \frac{\partial f}{\partial y} = 8x^2y \).
(b) A function \( g(x, y) \) satisfying the equations \( \frac{\partial f}{\partial x} = \frac{\partial f}{\partial y} = 2xy \).

4. Use the appropriate version of the chain rule to compute the following:
(a) \( dw/dt \) at \( t = 3 \), where \( w = \ln(x^2 + y^2 + z^2) \), \( x = \cos t \), \( y = \sin t \), and \( z = 4\sqrt{t} \).
(b) \( \partial z/\partial u \) and \( \partial z/\partial v \), where \( z = xy \), \( x = u \cos v \), and \( y = u \sin v \).

5. Suppose a duck is swimming around in a circle, with position given by \( x = \cos t \) and \( y = \sin t \).
Suppose that the water temperature is given by \( T = x^2e^y - xy^3 \). Find the rate of change in temperature that the duck experiences as it passes through the point \( (1/\sqrt{2}, -1/\sqrt{2}) \).
6. Compute the following using implicit differentiation:

(a) \( \frac{\partial y}{\partial z} \) if \( e^{yz} - x^2 z \ln y = \pi \).
(b) \( \frac{dy}{dx} \) if \( F(x, y, x^2 - y^2) = 0 \).

7. The surface plot \( z = f(x, y) \) and the contour diagram are shown:

Look at the point \((2, 2)\). At this point, find the sign (positive or negative) of each of the following quantities:

- \( \frac{\partial f}{\partial x} \)
- \( \frac{\partial f}{\partial y} \)
- \( \frac{\partial^2 f}{\partial x^2} \)
- \( \frac{\partial^2 f}{\partial y^2} \)
- \( \frac{\partial^2 f}{\partial x \partial y} \)

8. Find the equation of the tangent plane to \( z = \sqrt{xy} \) at the point \((1, 1, 1)\).

9. You have three resistors labeled 10Ω, 20Ω and 30Ω. Each of the resistances is guaranteed accurate to within 1%.

   (a) You connect the resistors in series, hoping to get a resistance of 6000Ω. Use differentials to estimate the maximum error in the resistance.

   (b) You connect the resistors in parallel, hoping to get a resistance of \( \frac{60}{11} \)Ω. Use differentials to estimate the maximum error in the resistance.