Notes on a Basic Business Problem
MATH 104 and MATH 184
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This simple problem will introduce you to the basic ideas of revenue, cost, profit, and demand. These are not so hard, but you need to build a vocabulary so that you can use these terms properly without second-guessing yourself.

Demand can be a difficult concept when you first meet it. Demand is the relationship between the price of an item and the number of units that will sell at that price. It is a sociological relationship in that it is rooted in the behaviour of consumers. It is a basic principal of economics that the demand relationship has the characteristic that an increase in price will lead to a decrease in demand. The simplest such relationship is a linear one, and in the first problem we present, we will use a linear demand relationship. In general, demand relationships are non-linear. In practice, determining the "real" demand relationship for a product is non-trivial, but in this course you will always be given this relationship, or enough information to determine it. (You may wish to consider how realistic some of the demand relationships that we give you are.)

It is usual to use the variables p for the price of a unit, and q for the quantity demanded.

In the problem below, we will plot the demand relationship on the (q, p)-plane and treat p as a function of q. However, it is important to note that q is not really an *independent* variable in these problems (except mathematically). Moreover, although the producer has the ability to set the price p, the demand relationship is NOT in control of the producer, so setting p determines how many items q will be sold. (In the strictest sense, we are in the situation of a *monopoly producer*, but there is not need to dwell on this in this course.)

Many (but not all!) of you are taking a first year economics course, so you will be learning these terms in that course as well. You will discover that there will be differences over the term between the presentation in this course and that in economics; mostly this arises because first year economics courses do not have a calculus pre-requisite. Usually these differences come from either conventions or from choices about which quantities are studied in problems. These can make for some interesting points of discussion in class.

We will introduce a basic business problem as an example. Note that some brief notes on the business concepts in this problem appear after it in these notes. I'd like to thank my summer student Patrick Chan, a senior BCOM student, for his assistance in preparing these later notes.

A Basic Business Problem

Opple Inc. is the only manufacturer of the popular oPad. Opple estimates that when the price of the oPad is \$200, then the weekly demand for it is 5000 units. For every \$1 increase in the price, the weekly demand decreases by 50 units. Assume that the fixed costs of production on a weekly basis are \$100 000, and the variable costs of production are \$75 per unit.

(a) Find the linear demand equation for the oPad. Use the notation p for the unit price and q for the weekly demand.

ANS : $p = -\frac{1}{50}q + 300$

(b) Find the weekly cost function, $C = C(q)$, for producing q oPads per week. Note that $C(q)$ is a linear function.

ANS : $C(q) = 100000 + 75q$

(c) Find the weekly revenue function, $R = R(q)$. Note that $R(q)$ is a quadratic function.

ANS: $R(q) = p \cdot q = q\left(300 - \frac{1}{50}q\right)$.

Note that this revenue relationship "Revenue = price per unit times quantity demanded" is central in your studies in business. While it looks simple, we will run into situations throughout the term where this relationship will produce counterintuitive results for you because p and q are related by a demand relationship. For example, sometimes increasing the price will cause a decrease in revenue.

(d) The *break-even* points are where Cost equals Revenue; that is, where $C(q) = R(q)$. Find the break-even points for the oPad.

ANS: Solve the quadratic you get by setting $R = C$.

(e) On the same set of axes, sketch graphs of $C = C(q)$ and $R = R(q)$ and use these graphs to help you explain why there are two break-even points.

ANS: Make a reasonable sketch, with appropriate points labelled.

(f) *Profit* is defined as Revenue minus Cost: $P(q) = R(q) - C(q)$. Find the profit function $P(q)$. Note that it is a quadratic function.

ANS: This is a simple calculation.

(g) Graph $P = P(q)$ on the same axes as you sketched the graphs of $C(q)$ and $R(q)$. On this graph, indicate the regions of profit ($P(q) > 0$) and loss ($P(q) < 0$).

ANS: The easiest way to indicate it is to point out the interval on the q-axis where $P > 0$.

(h) How should Opple Inc. operate in order to maximize the weekly profit $P = P(q)$? Use mathematics in your explanation.

ANS: Find the vertex of the Profit quadratic. One might also use calculus to solve this problem (which is a bit of overkill for this quadratic function). Those of you in Math 104 will know this procedure, at least at this elementary level. in Math 184, you will learn about this maximization procedure in more detail later in the course, so in the meantime, try to understand it graphically. When we look at more complicated demand relationships, we will need a procedure that will work on other functions in addition to quadratics. Fortunately, calculus will provide us with such a procedure.