

MATH 104: Week 9 and 10 Fresh Sheet

October 31, 2017

Learning Goals

There is a midterm in Week 9. As well, in Week 10 the MWF classes will not have class on Monday because of the holiday on November 13th.

This week we cover the first and second derivative tests and curve sketching. This material is in Section 3.6 of the Course Notes. This material will include another look at limits at infinity, infinite limits, and a first look at asymptotes.

The specific learning goals for this section are that by the end of Week 10, you should be able to:

1. use calculus to sketch a graph of a given function. Specifically, this means you will be able to
 - (a) explain how the first derivative of a function determines where the function is increasing and decreasing and apply this to specific functions to determine their intervals of increase and decrease;
 - (b) use the first derivative test to identify local maxima and minima;
 - (c) explain how the second derivative of a function determines concavity and apply this to specific functions to determine where they are concave up and concave down, and to identify inflection points;
 - (d) use the second derivative test to classify local maxima and minima;
 - (e) identify any asymptotic behaviours a function may have: vertical asymptotes, horizontal asymptotes, and oblique or slant asymptotes;

Food for Thought As You Study This Week

1. The material in 3.6 is presented using the viewpoint “what derivatives tell us.” It is crucial to spend time this week making links between the calculations you do and the intuitive aspects of drawing sketches of functions to help you interpret your results. I do recommend that you use a graphing tool (e.g. www.wolframalpha.com) so that students can see how they can check your answers by viewing the final graphs. It will be worthwhile to show them that wolframalpha also has some behaviours that give them unexpected results for functions like $f(x) = x^{2/3}$.
2. It is useful to master “good bookkeeping” techniques for the first derivative test. The kinds of figures that encode the first derivative test (e.g. see the table in Example 3.6.2) are very useful to students to track their results. They are also a good way to present the information clearly and precisely. There are other forms of such tables, but the result is the same. Students who do not track their thinking and express it through these sorts of tables or diagrams tend to get lost and lose many marks on exams.
3. FOR THOSE LOOKING FOR MASTERY OF DEEPER CONCEPTS: Inflection points are worth thinking about carefully. The Course Note’s definition of an inflection point is not consistent with the actual geometric definition of such points. For example, there is an inflection point in the curve $y = x^{1/3}$ at $x = 0$ even though neither $f'(0)$ nor $f''(0)$ are defined. There is a vertical tangent line at this point and the curve does change concavity as you pass through this point. Looking at this as the graph of $x = y^3$ may help the students see this.

4. We expect you to be able to find vertical, horizontal, and oblique asymptotes.
5. The curve sketching checklist in section 3.6.5 is nicely presented. We encourage you to practice sketching many curves!