Learning Goals

We will cover implicit differentiation, section 2.11 of the Course Notes. We also will cover section 2.10 of the Course Notes, which is on the natural logarithm, though it also covers some basic ideas for general logarithms. Students will learn the technique of logarithmic differentiation, which will be important for Week 6 when we cover elasticity of demand and relative rates of change.

Note that Monday, October 9th is Thanksgiving and a holiday, so this is a short week for those teaching MWF.

The specific learning goals for this week are that by the end of the week and review homework, students should be able to:

1. explain what we mean by *implicit differentiation* and identify situations where they will use it;
2. carry out computations involving implicit differentiation;
3. find equations of tangent lines to graphs of implicitly defined functions;
4. find equations of normal lines to graphs of implicitly defined functions;
5. use the implicit differentiation to demonstrate the power rule for rational exponents;
6. work with the inverse properties of $e^x$ and $\ln x$;
7. use the derivatives of general logarithmic functions in computations;
8. use the derivatives of general exponential functions in computations;
9. use the technique of logarithmic differentiation.

Some Food for Thought as You Study This Week

1. Our approach to logarithmic and exponential functions will be to first focus on $e^x$ and $\ln x$. You can then use the inverse function relationship and implicit differentiation to find the derivative of $\ln x$ from the derivative of $e^x$. You can then approach general logarithms and exponentials.
2. One of your most important learning goals is to build a computational proficiency in computing derivatives of functions involving logarithms and exponentials.
3. Pay attention to logarithmic differentiation. Besides being a useful tool, in the future we will use this idea to explore some concepts in economics. One key point: you can define the relative rate of change of a function $f(x)$ as $f'(x)/f(x)$, which is equal to the derivative of $\ln(f(x))$ (when this is defined).
4. Some loci of points in the plane, like the unit circle $\{x^2 + y^2 = 1\}$, are not defined simply as the graph of a function $y = f(x)$. However, we can still find geometric tangent lines to these curves – at every point, in the case of the unit circle – and so implicit differentiation gives us a way of connecting this fact to using the derivative notion to define these tangent lines. As an interesting question: what sorts of things do you imagine happen when there is a self-intersection point, which is a point where the curve crosses itself?
5. What is the relationship between the slopes of the tangent line and the normal line at a point on a curve where they both exist?