Related Rates Problems Solutions

MATH 104

2017W

1. The workers in a union are concerned whether they are getting paid fairly or not. They are specifically concerned at the rate at which wages are increasing per year is lagging behind the rate of increase in the company's profit's per year. In order for the wage increase to be fair, the rate that the wage increases per year should be the same as the rate that the company's profit is increasing per year. Currently, the wage (L) is \$24.00 per hour on average for each worker. Determine whether this is fair or not given that the profit function is the following:

$$P = \frac{21}{100}L^3 - 4L^2.$$

SOLUTION

Need to derive the function implicitly with respect to time to find $\frac{\partial P}{\partial t}$ and $\frac{\partial L}{\partial t}$

$$\frac{\partial P}{\partial t} = \frac{63L^2}{100} \cdot \frac{\partial L}{\partial t} - 8L \cdot \frac{\partial L}{\partial t} \tag{1}$$

In order for wages to be fair $\frac{\partial P}{\partial t}$ and $\frac{\partial L}{\partial t}$ must equal to each other.

$$\frac{\partial P}{\partial t} = \frac{\partial L}{\partial t} \cdot \left(\frac{63L^2}{100} - 8L\right) \tag{2}$$

We can cross out $\frac{\partial P}{\partial t}$ and $\frac{\partial L}{\partial t}$ since they equal to each other. Now we will have

$$1 = \left(\frac{63L^2}{100} - 8L\right). \tag{3}$$

L can be solved using the quadratics equation and we will get

$$\boxed{L = \$12.82.} \tag{4}$$

2. The monthly revenue R (in dollars) of a telephone polling service is related to the number x of completed responses by the function

$$R(x) = -13450 + 60\sqrt{6x^2 + 20x},$$

where $0 \leq x \leq 1500$. If the number of completed responses is increasing at the rate of 10 forms per month, find the rate at which the monthly revenue is changing when $x = 700$.

SOLUTION

$$\frac{\partial R}{\partial t} = 60 \cdot \frac{1}{2} \cdot \frac{1}{\sqrt{6x^2 + 20x}} \cdot \left(12x\frac{\partial x}{\partial t} + 20\frac{\partial x}{\partial t}\right) \tag{5}$$

We know that $\frac{\partial x}{\partial t} = 10$ and want to find what $\frac{\partial R}{\partial t}$ is when $x = 700$.

$$\frac{\partial R}{\partial t} = 60 \cdot \frac{1}{2} \cdot \frac{1}{\sqrt{6 \cdot 700^2 + 20 \cdot 700}} \cdot (12 \cdot 700 \cdot 10 + 20 \cdot 10) \tag{6}$$

$$\boxed{\frac{\partial R}{\partial t} = \$1469.70} \tag{7}$$

3. The owner of Cazio Watches Co. wants to predict how interest rates effect monthly sales. If the current interest rate r is 4% and the monthly change in interest rate is 0.8%, what is the change in sales per month if sales are determined by the function:

$$S = \frac{150000}{\sqrt{r^2 + 5}} - \frac{4900r^2}{3},$$

where S is in hundreds of dollars?

SOLUTION

We know: $r = 0.04$ and $\frac{dr}{dt} = 0.008$
Want: $\frac{dS}{dt}$

$$\frac{dS}{dt} = -75000(r^2 + 5)^{-\frac{3}{2}} \cdot 2r\frac{dr}{dt} - \frac{9800r}{3} \cdot \frac{dr}{dt} \tag{8}$$

Plugging in what we know, we will get

$$\frac{dS}{dt} = -5.3365 \text{ hundreds of dollars} = -\$533.65 \tag{9}$$

There will be a decrease of \$533.65 in sales revenue for the upcoming month.

4. General Farms Cereal makes q thousand packs of Fruit Loops Cereal in the marketplace each week when the wholesale price is $\$p$ per box. The relationship between x and p is governed by the supply equation

$$6q^2 - 5qp + 2p^3 = 5.$$

How fast is the supply of cereals changing when the price per box is \$6.50, the quantity supplied is 10,000 boxes, and the whole sale price per box is increasing at the rate of \$0.10 per box box each week?

SOLUTION

We know:
$p = 6.50$, $x = 10$, $\frac{dp}{dt} = 0.1$.

$$\frac{d}{dt}(6x^2) - \frac{d}{dt}(5xp) + \frac{d}{dt}(2p^3) = \frac{d}{dt}(5) \tag{10}$$

$$12x \cdot \frac{dx}{dt} - 5p \cdot \frac{dx}{dt} - 5x \cdot \frac{dp}{dt} + 6p^2 \cdot \frac{dp}{dt} = 0 \tag{11}$$

Plugging in the variables we know, we will get:

$$12(10) \cdot \frac{dx}{dt} - 5(6.50) \cdot \frac{dx}{dt} - 5(10)(0.1) + 6(6.50)^2(0.1) = 0. \tag{12}$$

Isolating for $\frac{dx}{dt}$, we will get:

$$\boxed{\frac{dx}{dt} = -0.23} \tag{13}$$

The supply of cereals are decreasing at a rate of 230 boxes per week when the price per box is \$6.50, quantity supplied is 10,000 boxes, and the whole sale price per box is increasing at the rate of \$0.10 per box each week.

5. It is estimated that the number of housing starts, $N(t)$ (in units of a million), over the next 5 years is related to the mortgage rate $r(t)$ (percent per year) by the equation

$$9N^2 + r = 36.$$

What is the rate of change of the number of housing starts with respect to time when the mortgage rate is 6% per year and is increasing at the rate of 0.25% per year?

SOLUTION

Given $r = 6$, and $\frac{dr}{dt} = 0.25$.

We want to find $\frac{dN}{dt}$

Derive the equation implicity with respect to t.

$$\frac{d}{dt}(9N^2) + \frac{d}{dt}(r) + \frac{d}{dt}(36) \tag{14}$$

$$18N \cdot \frac{dN}{dt} + \frac{dr}{dt} = 0 \tag{15}$$

We need to know N in order to solve (15) to find $\frac{dN}{dt}$. We will use the equation given in the question to solve for N.

Since we know that $r = 6$, we can solve for N.

$$9N^2 + 6 = 36 \tag{16}$$

$$N = \sqrt{\frac{10}{3}} \tag{17}$$

The negative root is ignored because we cannot have negative number of housing starts.

Plugging in what we know in to equation (15), we will get

$$18\sqrt{\frac{10}{3}} \cdot \frac{dN}{dt} + 0.25 = 0 \tag{18}$$

$$\boxed{\frac{dN}{dt} = -0.007607} \tag{19}$$

Thus, at the instant of time under consideration, the number of housing starts is decreasing at the rate of 7606 units per year.