This simple problem will introduce students to the basic ideas of revenue, cost, profit, and demand. These are not so hard, but they need to build this vocabulary so that they can use these terms properly without second-guessing themselves.

Demand can be a difficult concept for students. Demand is the relationship between the price of an item and the number of units that will sell at that price. It is a sociological relationship in that it is rooted in the behaviour of consumers. It is a basic principal of economics that the demand relationship has the characteristic that an increase in price will lead to a decrease in demand. The simplest such relationship is a linear one, and in the first problem we present, we will use a linear demand relationship. In general, demand relationships are non-linear.

It is usual to use the variables $p$ for the price of a unit, and $q$ for the quantity demanded. Please do this consistently throughout the term.

In the problem below, we will plot the demand relationship on the $(q, p)$-plane and treat $p$ as a function of $q$. However, it is important to note that $q$ is not really an independent variable in these problems (except mathematically). Moreover, although the producer has the ability to set the price $p$, the demand relationship is NOT in control of the producer, so setting $p$ determines how many items $q$ will be sold. (In the strictest sense, we are in the situation of a monopoly producer, but there is not need to dwell on this in this course.)

Many (but not all!) of your students will be taking a first year economics course, so will be learning these terms in that course as well. You will discover that there will be differences over the term between your presentation and natural inclinations and those of our colleagues in Economics. Usually these come from either conventions or from choices about which quantities are studied in problems. These can make for some interesting points of discussion in your class.

A Business Problem

Opple Inc. is the only manufacturer of the popular oPad. Opple estimates that when the price of the oPad is $200, then the weekly demand for it is 5000 units. For every $1 increase in the price, the weekly demand decreases by 50 units. Assume that the fixed costs of production on a weekly basis are $100 000, and the variable costs of production are $75 per unit.

(a) Find the linear demand equation for the oPad. Use the notation $p$ for the unit price and $q$ for the weekly demand.

\[ p = -\frac{1}{50}q + 300 \]

ANS : $p = -\frac{1}{50}q + 300$

(b) Find the weekly cost function, $C = C(q)$, for producing $q$ oPads per week. Note that $C(q)$ is a linear function.

\[ C(q) = 100000 + 75q \]

ANS : $C(q) = 100000 + 75q$
(c) Find the weekly revenue function, \( R = R(q) \). Note that \( R(q) \) is a quadratic function.

ANS: \( R(q) = p \cdot q = q(300 - \frac{1}{50}q) \).

Note that this revenue relationship “Revenue = price per unit times quantity demanded” is central in our students’ studies in business. While it looks simple to us, we will run into situations throughout the term where this relationship will produce counterintuitive results for students because \( p \) and \( q \) are related by a demand relationship. For example, sometimes increasing the price will cause a decrease in revenue.

(d) The break-even points are where Cost equals Revenue; that is, where \( C(q) = R(q) \). Find the break-even points for the oPad.

ANS: Solve the quadratic you get by setting \( R = C \).

(e) On the same set of axes, sketch graphs of \( C = C(q) \) and \( R = R(q) \) and use these graphs to help you explain why there are two break-even points.

ANS: Make a reasonable sketch, with appropriate points labelled.

(f) Profit is defined as Revenue minus Cost: \( P(q) = R(q) - C(q) \). Find the profit function \( P(q) \). Note that it is a quadratic function.

ANS: Simple calculation.

(g) Graph \( P = P(q) \) on the same axes as you sketched the graphs of \( C(q) \) and \( R(q) \). On this graph, indicate the regions of profit \( (P(q) > 0) \) and loss \( (P(q) < 0) \).

ANS: The easiest way to indicate it is to point out the interval on the \( q \)-axis where \( P > 0 \).

(h) How should Opple Inc. operate in order to maximize the weekly profit \( P = P(q) \)? Use mathematics in your explanation.

ANS: Find the vertex of the Profit quadratic. This is a good place to indicate the possible use of calculus to solve this problem. In Math 104, the students will know this procedure, at least at this elementary level. In Math 184, you will need to motivate it. You can then point out that when we look at more complicated demand relationships, we will need a procedure that will work on other functions in addition to quadratics.