**Extra Business Problems #1**
MATH 104
September 26, 2017

1. Consider the cost function $C(q) = 6q^2 + 14q + 18$ (in thousands of dollars).

   (a) What is the marginal cost at production level $q = 5$?
   *solution:* $C^{'}(q) = 12q + 14$, so $C^{'}(5) = 74$.

   (b) Estimate the cost of raising the production level from $q = 5$ to $q = 5.25$.
   *solution:* $C^{'}(5) \times (5.25 - 5) = 18.5$

   (c) Let $R(q) = -q^2 + 37q + 38$ denote the revenue in thousands of dollars from the production of $q$ units. What is the break-even point?
   *solution:*
   $$C(q) = R(q) \Rightarrow (7q + 5)(q - 4) = 0$$
   Since $q \geq 0$, the break-even point is $q = 4$.

   (d) Compute and compare the marginal revenue and marginal cost at the break-even point. Should the company increase production beyond the break-even point? Justify your answer using marginals.
   *solution:*
   $$C^{'}(q) = 12q + 14, \quad R^{'}(q) = -2q + 37$$
   marginal revenue $R^{'}(4) = 29$, marginal cost $C^{'}(4) = 62 > R^{'}(4)$, so the company should not increase production beyond break-even point because it will decrease profit.

2. When EZ Electronics Company sells surge protectors at $50 a piece, they produce and sell 3000 of them per month. For every $1 increase in price, the number of surge protectors they sell decreases by 15. Assume that the fixed production costs are $50,000 and the variable costs are $30 per surge protector.

   (a) Find the linear demand function $q = D(p)$, where $p$ is a price of a unit and $q$ is the number of surge protectors made and sold. [Hint: The point $(p, q) = (50, 3000)$ must lie on this line.]
   *solution:*
   $$q - 3000 = -15(p - 50) \Rightarrow D(p) = -15p + 3750$$

   (b) Find the cost function $C(q)$ as a function of $q$, and then express it as a function of $p$.
   *solution:*
   $$C(q) = 30q + 50\,000$$
   $$C(p) = -450p + 162\,500$$

(c) Find the revenue function $R(q)$ as a function of $q$, and then express it as a function of $p$.

*solution:*

$$R(q) = pq = -\frac{1}{15}q^2 + 250q$$
$$R(p) = pq = -15p^2 + 3750p$$

(d) Find the marginal profit, $MP(p)$, with respect to $p$.

*solution:*

$$MP(p) = R^{'}(p) - C^{'}(p) = -30p + 4200$$

(e) Find the *break-even points.* Give both the price $p$ and quantity $q$ at each of these points.

*solution:*

$$C(p) = R(p) \Rightarrow 15p^2 - 4200p + 162\,500 = 0$$
$$p = \frac{4200 \pm \sqrt{4200^2 - 4 \cdot 15 \cdot 162500}}{30} \approx 46.4 \text{ or } 233.6$$

break-even points: $(p, q) = (46.4, 3054)$ or $(233.6, 246)$.

(f) If EZ Electronics Company is operating at the higher break-even point, should it increase or decrease the price of its surge protectors to increase its profits? Explain your answer.

*solution:* $MP(p = 233.6) < 0$. Decrease the price to increase profits.

3. Tellyou Phone Company produces cell phones based on a cost function given by $C(q) = 300q + 40000$, where $q$ is the quantity of cell phones made and sold each day. Suppose that their accountants have determined that their revenues are given by the function $R(q) = 800q - q^2$.

(a) What are the *break-even* points for the Tellyou Phone Company?

*solution:* $q = 100$ or $400$.

$$C(q) = R(q) \Rightarrow (q - 100)(q - 400) = 0$$

(b) A company is *profitable* if the profit function is positive. Sketch the profit function for the Tellyou Phone Company. What is the range on the number of cell phones the Tellyou Phone Company should make and sell in order to be profitable? Give an exact answer and also indicate this range on your sketch.

*solution:* the range is $(100, 400)$.

$$P(q) = R(q) - C(q) = -q^2 + 500q - 40\,000$$

(c) If Tellyou Phone Company currently is producing 300 cell phones per day, will increasing their production increase their profit? Explain your answer mathematically using marginals.

*solution:*

$$MP(q) = -2q + 500$$

$MP(300) = -100 < 0$, so it will not increase the profit.

(d) What is the demand function for the Tellyou Phone Company?

$$pq = R(q) = 800q - q^2 \Rightarrow p = 800 - q$$

$$q = D(p) = -p + 800$$

4. One of the reasons that companies may lower costs in the long-run is they benefit from *learning-by-doing*, which is the productive skills and knowledge of better ways to produce that workers and managers gain from experience. Suppose that a company's average cost is $AC(q) = \alpha q^{\beta}$, where $\alpha > 0$, and $q$ is the company's output. How can you interpret $\alpha$ in terms of the business? (*Hint:* Suppose that $q = 1$.) What sign must $\beta$ have if there is learning-by-doing? What happens to the average cost as $q$ gets large Draw the average cost curve as a function of output $q$ for a particular set of values $\alpha$ and $\beta$.

   *solution:* $\alpha = AC(1)$, so $\alpha$ is the company's average cost at 1 output. Assuming learning-by-doing, the average cost $AC$ decreases as output $q$ increases, hence $\beta < 0$. $AC(q)$ is positive, monotonically decreasing and $AC(q) \to 0$ as $q \to \infty$.

5. Mei works in a flower shop, where she produces 10 floral arrangements per hour. She is paid \$10 per hour for the first eight hours she works, and then \$15 per hour for each additional hour she works.

   (a) What is the flower shop's labour cost function as a function of the number of floral arrangements produced based on Mei's work?

   *solution:* Mei produces 80 floral arrangements in the first 8 hours, and the labour cost is \$1/arrangement, $C(q) = q$. After 8 hours, labour cost is \$1.5/arrangement, $C(q) = 80 + 1.5(q - 80)$. So

$$C(q) = \begin{cases} q & q \leq 80 \\ 1.5q - 40 & q > 80 \end{cases}$$

   (b) What is the average labour cost of producing $q$ floral arrangements?

   *solution:*

$$AC(q) = \frac{C(q)}{q} = \begin{cases} 1 & q \leq 80 \\ 1.5 - 40q^{-1} & q > 80 \end{cases}$$

   (c) What is the marginal labour cost at the production level of $q$ floral arrangements?

   *solution:*

$$MC(q) = \frac{\Delta C(q)}{\Delta q} = \begin{cases} 1 & q < 80 \\ 1.5 & q \geq 80 \end{cases}$$

6. The FlipIt Novelty Company estimates that it can sell 2000 pet rocks per month if it sets a unit price of \$5.00. FlipIt estimates that for each \$0.20 decrease in price per unit, its sales of pet rocks will increase by 200 per month.

   (a) Find the demand function for pet rocks.
   *solution:*
   $$q - 2000 = -\frac{200}{0.2}(p-5) \Rightarrow q = D(p) = -1000p + 7000$$

   (b) Find the revenue function for pet rocks.
   *solution:*
   $$R(p) = pq = -1000p^2 + 7000p$$

   (c) Find the number of pet rocks FlipIt should sell per month in order to maximize the monthly revenue.
   *solution:*
   $$R(p) = -1000(p-3.5)^2 + 12250$$
   maximum revenue is reached when $p = 3.5$, so FlipIt should sell $D(3.5) = 3500$ pet rocks per month.

   (d) What is the maximum monthly revenue from the sales of pet rocks?
   *solution:* $R_{\text{max}} = \$12\,250$.