**Continuous Compound Interest Problems**
MATH 104
October 10, 2017

1. Find the present value of \$5000 to be received in 2 years if the money can be invested at 12% annual interest rate compounded continuously.

   *Solution:* $\$5000e^{12\% \times 2} = \$6356.25$

2. An investment earns at an annual interest rate of 4% compounded continuously. How fast is the investment growing when its value is \$10 000?

   *Solution:* $A(t) = Pe^{rt}$, $A'(t) = Pre^{rt} = rA(t)$. So when $A(t)$ is \$10 000, $A'(t) = 4\% \times 10\,000 = 400$.

3. One thousand dollars is deposited in a savings account at 6% annual interest rate compounded continuously. How many years are required for the balance in the account to reach \$2500?

   *Solution:* After $t$ years, the balance will be $A(t) = 1000e^{0.06t}$. So

$$\begin{aligned} 1000e^{0.06t} &= 2500 \\ e^{0.06t} &= 2.5 \\ 0.06t &= \ln 2.5 \\ t &= \ln 2.5/0.06 \approx 15.27 \end{aligned}$$

4. In a certain neighbourhood of Vancouver, property values tripled from 2001 to 2011. If this trend continues, when will property values be five times their 2001 level? Assume property values behave as if the annual investment rate is compounded continuously.

   *Solution:* Let $P$ be the property values in 2001 and $r$ the annual interest rate. In 2011 property value triples means

$$Pe^{10r} = 3P \Rightarrow e^r = \sqrt[10]{3}$$

   If $t$ years after 2001 property values would be $5P$, then

$$5P = Pe^{rt} = (\sqrt[10]{3})^t P \Rightarrow t = \ln 5/\ln \sqrt[10]{3} \approx 14.65$$

5. Suppose that the present value of \$1000 to be received in 5 years is \$550. What rate of interest, compounded continuously, was used to compute this present value?

   *Solution:* Let $r$ be the rate of interest, then

$$\begin{aligned} 1000e^{5r} &= 550 \\ r &= \frac{1}{5}\ln 0.55 \approx -11.96\% \end{aligned}$$

6. Investment A is worth \$70 thousand, and is growing at a rate of 13% per year compounded continuously. Investment B is worth \$60 thousand is growing at a rate of 14% per year compounded continuously. After how many years will the two investments have the same value?

   *Solution:* Assume it is after $t$ years, then

$$\begin{aligned} 70e^{0.13t} &= 60e^{0.14t} \\ e^{-0.01t} &= \frac{6}{7} \\ t &= \frac{\ln(6/7)}{-0.01} \approx 15.42 \end{aligned}$$