

NAME: SOLUTIONS

STUDENT NUMBER:

**Quiz # 4**

MATH 101 Section 205 (MacLean)

Friday, March 2, 2007

1. Find the partial fraction decomposition of  $\frac{2}{x^2 + 3x - 4}$   
First, factor the denominator as  $x^2 + 3x - 4 = (x + 4)(x - 1)$   
Then,

$$\begin{aligned}\frac{2}{x^2 + 3x - 4} &= \frac{A}{x + 4} + \frac{B}{x - 1} \\ &= \frac{A(x - 1) + B(x + 4)}{(x - 1)(x + 4)}\end{aligned}$$

Now, we have the following system of equations

$$0 = A + B, 2 = -A + 4B$$

Solving this gives

$$A = -\frac{2}{5}, B = \frac{2}{5}$$

2. Evaluate  $\int_0^1 \frac{2x + 3}{(x + 1)^2} dx$ .

Use partial fractions on the integrand first

$$\begin{aligned}\frac{2x + 3}{(x + 1)^2} &= \frac{A}{x + 1} + \frac{B}{(x + 1)^2} \\ &= \frac{A(x + 1) + B}{(x + 1)^2}\end{aligned}$$

This yields the system of equations

$$\begin{aligned}2 &= A, \\ 3 &= A + B.\end{aligned}$$

Solving this gives

$$A = 2, B = 1.$$

Therefore,

$$\begin{aligned}\int_0^1 \frac{2x+3}{(x+1)^2} dx &= \int_0^1 \frac{2}{x+1} dx + \int_0^1 \frac{1}{(x+1)^2} \\ &= 2 \ln|x+1| \Big|_0^1 - \frac{1}{x+1} \Big|_0^1 \\ &= 2(\ln 2 - \ln 1) + 1 - \frac{1}{2} \\ &= 2 \ln 2 + \frac{1}{2}.\end{aligned}$$