

## MATH 101: Week 8 Learning Goals

March 2, 2014

This week one main topics: separable differential equations (section 9.3). Note that we are not working with the part of 9.3 on orthogonal trajectories.

### Learning Goals

The specific learning goals for this week are that by the end of the week and review homework, you should be able to:

1. Construct a **first-order differential equation** with appropriate **initial condition** that can be used to model a quantity described in a problem by finding a relationship between the quantity and its derivative. Find a function that solves a **separable differential equation** with or without an initial condition.

### Potential Learning Approaches and Issues

1. In Math 100, you met your first differential equation:  $dx/dt = kx$ , which has a general solution  $x(t) = ce^{kt}$ , where  $c$  is an arbitrary constant. If you consider an extra piece of information, say an initial value, say  $x(0) = x_0$ , the  $c$  becomes determined, so  $x(t) = x_0e^{kt}$ . Now that you know how to integrate, rather than “guess” the solution to this differential equation, you can find it by a computation via differentials:

$$\begin{aligned}\frac{dx}{x} &= k dt, \\ \implies \int \frac{dx}{x} &= \int k dt, \\ \implies \ln|x| &= kt + C, \\ \implies |x| &= e^{kt+C}, \\ \implies x(t) &= e^C e^{kt}.\end{aligned}$$

Again, if you have an initial value, say  $x(0) = x_0$  then  $x(t) = x_0e^{kt}$ .

2. The kinds of differential equations in section 9.3 are called *separable differential equations*, meaning that you can arrange to do a computation using the same ideas as we used for  $dx/dt = kx$ . In the more general equation, we have  $dy/dx = f(x)g(y)$  – that is, we are able to write the derivative  $dy/dx$  as a product that completely separates the dependencies on  $x$  and  $y$ . This means that you start with

$$\begin{aligned}\frac{dx}{f(x)} &= \frac{dy}{g(y)}, \\ \implies \int \frac{dx}{f(x)} &= \int \frac{dy}{g(y)}.\end{aligned}$$

Thus, the barrier to finding functional forms for solutions to such differential equations is the ability to find antiderivatives. (Numerical methods are possible in cases where finding antiderivatives is not feasible.)

3. Again, initial-value problems arise. There are many basic applications of initial-value problems involving separable equations, such as looking at growth that is limited in some way (e.g. logistic growth), mixing problems, Newton's law of cooling problems, charging capacitor problems, circuits involving inductors, and so on.
4. It turns out that more general separable differential equations are very useful in considering more general problems, and make appearances in problems involving heat flow, wave motions, and the equation governing an electron in a hydrogen atom, for example. These applications are beyond MATH 101, but you may be introduced to them in your physics or chemistry courses. In general, such equations involve second derivatives.

## Suggested Problems

Section 9.3: 1, 3, 5, 8, 14, 15, 16, 19, 22, 33, 40, 42, 45, 54.