

## MATH 101: Week 5 Learning Goals

January 28, 2014

### Learning Goals

This week we study three techniques of integration: (1) trigonometric integrals, and (2) trigonometric substitutions, and (3) partial fractions. In many ways, the techniques involve transforming integrals using identities, some algebra, or changes of variable.

The specific learning goals for this week are that by the end of the week and review homework, you should be able to:

1. use trigonometric identities to find antiderivatives of functions that are simple combinations of the basic trigonometric functions;
2. use changes of variable involving trigonometric functions to find antiderivatives of functions;
3. use the algebraic notion of partial fraction decomposition to find the antiderivatives of rational functions (polynomials divided by polynomials).

### Potential Learning Approaches and Issues

1. The focus of this week is developing several techniques to find antiderivatives of functions. The key to keeping the different techniques straight is to think carefully about the basic principles upon which they are built. The only way to develop facility with these techniques is to *do many, many examples* yourself. You can use something like wolframalpha to check your answers.
2. Integrals involving trigonometric functions are generally attacked using various trigonometric identities. In reality, we will use only a handful of these, but you will want to memorize them if you don't already know them. These include the basic  $\sin^2 x + \cos^2 x = 1$  and its variants, as well as double-angle formulas such as  $\sin 2x = 2 \sin x \cos x$ , and half-angle formulas such as  $\sin^2 x = (1 - \cos 2x)/2$ .
3. We will not ask you to deal with combinations of sine and cosine such as  $\sin mx \cos nx$ , etc.
4. A variant of the substitution method we have already studied is to make a substitution of the form  $x = f(u)$ , with  $dx = f'(u)du$ . In section 7.3, the focus is on using trigonometric functions. For example, if the integrand contains a form like  $\sqrt{1 - x^2}$ , then it is likely that setting  $x = \sin u$  will help make things simpler. The point is that trigonometric identities, such as  $1 - \sin^2 x = \cos^2 x$  in this case, transform sums or differences of squares into perfect squares.
5. Ignore inverse secant and inverse hyperbolic functions.
6. It is worthwhile reviewing the basic properties of trigonometric functions. In particular, you will want to think about their inverses and the notion of principal branches of inverse trigonometric functions. This will help you track the appropriate signs in applying the change of variables formula to definite integrals. For the most part, things work out as you expect, but not always.
7. The method of partial fractions is based in a bit of algebra about rational function. Recall that rational functions are ones that are obtained by dividing one polynomial by another polynomial. Partial fractions is effectively a method of reversing the adding of fractions by finding a “common denominator.” The algebra may seem mysterious, though in simple cases, you can track the

calculation to see that it is correct. The actual algebraic theorem that makes this work is beyond MATH 101. However, the method itself is not so difficult. At one stage, it does involve solving several linear equations in several variables, so you may wish to dust off that bit of pre-calculus knowledge.

8. The first step in the partial fractions method involves factoring the denominator of the rational integrand. You will use a variant of the Fundamental Theorem of Algebra that says every polynomial of degree 2 or higher with real numbers as coefficients can be fully factored into a constant times factors that are linear (which may be repeated) or at most quadratic (which may be repeated). Note that we are factoring over the real numbers, and not the rational numbers, so some of the things you know from high school do not apply: for example,  $x^4 + 1$  has degree 4 and is not factorable over the rational numbers. However, over the real numbers it factors:  $x^4 + 1 = (x^2 - \sqrt{2}x + 1)(x^2 + \sqrt{2}x + 1)$ . You can check using the discriminant for the quadratic formula that neither of the two quadratics factors further over real numbers.
9. The method of partial fractions can be used when you have ratios of polynomials in other functions. For example, if you wish to find the antiderivative of

$$\frac{\cos(\theta)}{1 - \sin^2(\theta)}$$

, you can use the fact that

$$\frac{1}{1 - \sin^2(\theta)} = \frac{1/2}{1 - \sin(\theta)} + \frac{1/2}{1 + \sin(\theta)}.$$

10. We will ignore Case IV: Repeated quadratic factors, in section 7.4.

## Suggested Problems

**Suggested Problems:** Each week, I will post suggested problems on the main Math 101 website. These are in addition to the formal assignments you will do through WebWork.

This week, all suggested problems will be from the text:

Section 7.2: Do as many of the integrals in this section as you can. 56, 66, 70\*.

Section 7.3: Do as many of the integrals in this section as you can. 35, 38, 39, 42

Section 7.4: Do as many of the integrals in this section as you can. 65, 66, 72. (Number 59 is a nice problem if you are keen.)