

MATH 101: Week 2 Learning Goals

January 14, 2014

Learning Goals

This week we introduce the Fundamental Theorem of Calculus and begin our program of studying the techniques of integration by introducing the idea that a change of variable (substitution) may sometimes be used to calculate definite integrals.

The specific learning goals for this week are that by the end of the week and review homework, you should be able to:

1. correctly state the fundamental theorem of calculus and use it to explain the relationship between integration and differentiation;
2. compute definite integrals by using the **fundamental theorem of calculus** in the case where the antiderivative is elementary;
3. recognize functions that are given as definite integrals with variable upper and lower limits and find their derivatives;
4. understand the **net change** as the definite integral of a rate of change and discuss the relationship of this result to the fundamental theorem of calculus
5. recognize when a substitution may be used to simplify the calculation of a definite integral and correctly transform a definite integral using that substitution.

Potential Learning Approaches and Issues

1. Many of you will have prior knowledge of what Stewart has called the Fundamental Theorem of Calculus, Part II. You may find the idea of defining a function via integration quite mysterious at first, and may resist our attempts have you work with what Stewart calls the Fundamental Theorem of Calculus, Part I. In your study of this theorem, you should emphasize how Part I encapsulates the relationship between integration and differentiation by doing explicit examples in which you differentiate functions of the form

$$g(x) = \int_a^x f(t) dt.$$

2. It can be useful to use Part II, which may be more comfortable for you, to help you understand Part I, at least computationally. That is, explicitly compute $g(x)$, as above, in simple cases through antiderivatives and then differentiate the result. You will quickly learn that you can avoid the intermediate step if you seek the derivative of such a function.
3. Be sure to do examples that are variations of the theme in Example 4 of section 5.3: explore functions of the form

$$g(x) = \int_{a(x)}^{b(x)} f(t) dt.$$

Look at the problems for this section to find such examples.

4. Table 1 of section 5.4 should be noted as essentially a table of derivatives, rewritten.

5. It is worth thinking about the $+C$ used in this table (and elsewhere) and what it means.
6. It is worth noting that the Net Change Theorem in section 5.4 is a restatement of Part II of the Fundamental Theorem of Calculus. Do you see the relationship? (You may recognize that your grade 11/12 physics courses made heavy use of this relationship.) It may help to focus on two physical examples rather than going through the whole list of them that appear after the statement of the theorem.
7. One approach to substitution is from the perspective that change of variable is an important and useful idea more generally in mathematics. You might wish to explore the geometry of these substitutions. We change the way we measure distances on the real line when we make such a substitution, for example: if $u = g(x)$ then $du = g'(x) dx$. Here, the small “distance” dx is multiplied by a factor, $g'(x)$, that depends on x to give you the small “distance” du . For example, if $u = x^2$ then $du = 2x dx$. If we take x to be in $[1, 2]$, then very near the end point $x = 1$, the way we measure distance in u is essentially by doubling the distance we had in x ; whereas, very near the endpoint $x = 2$, the way we measure distance in u is essentially by multiplying the distance we had in x by 4. You might think about this mapping $u = x^2$ as if $[1, 2]$ is a piece of an elastic band that is stretched differently at different locations under this changes of variables.

Suggested Problems

Suggested Problems: Each week, I will post suggested problems on the main Math 101 website. These are in addition to the formal assignments you will do through WebWork.

This week, all suggested problems will be from the text:

Section 5.3: 3, 7, 10, 13, 28, 37, 43, 46, 57, 67, 77.

Section 5.4: 3, 6, 21 to 34, 39, 41, 49, 63.

Section 5.5: 2, 5, 14, 20, 21, 25 to 36, 60, 74, 79, 86, 90, 91.