1. Prove that the fourth power of an odd integer is expressible in the form $16n + 1$ for $n \in \mathbb{Z}$.

2. Define $a_n$ by $a_0 = 1, a_1 = 2, a_3 = 4$ and
   
   $$a_{n+2} = a_{n+1} + a_n + a_{n-1}, \quad \text{for } n \geq 1.$$ 

   Show that $a_n \leq 2^n$ for all $n \in \mathbb{N}$. 

3. If $F_n$ is the $n$th Fibonacci number, prove that
\[ F_{n+1}F_{n-1} - F_n^2 = (-1)^n. \]

4. Let $a$ and $n$ be positive integers with $a > 1$. Prove that, if $a^n + 1$ is prime, then $a$ is even and $n$ is a power of 2.
5 Show that if all three of $p, p + 2$ and $p + 4$ are prime, then $p = 3$.

6 Use the Euclidean algorithm to compute $(2059, 2581)$ and to express this quantity as a linear combination of 2059 and 2581.
7  Show that every nonzero integer can be uniquely expressed as
    \[ a_k 3^k + a_{k-1} 3^{k-1} + \cdots + a_1 3 + a_0 \]
where \( a_i \in \{-1, 0, 1\} \) and \( a_k \neq 0 \).

8  Prove that there are infinitely many primes of the shape \( 6n + 5 \).