The remainder of this page has been left blank for your workings.
Very short answer questions

1. [2 marks] Each part is worth 1 mark. Please write your answers in the boxes. **Marking scheme:** 1 for each correct, 0 otherwise

(a) Find \( \frac{dy}{dx} \), if \( y = \frac{1 + 3x}{5 + 2x} \).

**Answer:** \( \frac{13}{(5 + 2x)^2} \)

**Solution:** Quotient rule:

\[
\frac{d}{dx} \left[ \frac{1 + 3x}{5 + 2x} \right] = \frac{(5 + 2x) \cdot 3 - (1 + 3x) \cdot 2}{(5 + 2x)^2}.
\]

This simplifies to \( \frac{13}{(5 + 2x)^2} \), though simplification is not required.

(b) The following limit is equal to the derivative of a function at a point:

\[
\lim_{x \to 1} \frac{5^x - 5}{x - 1}
\]

Evaluate the limit. You can (and should) use differentiation rules, but not l’Hopital’s rule.

**Answer:** \( 5 \log(5) \)

**Solution:** We compare the given limit with the following definition:

\[
f'(a) = \lim_{x \to a} \frac{f(x) - f(a)}{x - a}.
\]

Substituting \( f(x) = 5^x \) and \( a = 1 \) into this definition, we obtain the limit in question. Thus, the limit in question is equal to \( f'(1) = 5^1 \cdot \log(5) = 5 \log(5) \). (NOTE: For us, \( \log(x) \) is the same as \( \ln(x) \). Writing \( \ln(x) \) is fine (for now).)
Short answer questions — you must show your work

2. [4 marks] Each part is worth 2 marks.

(a) Let \( h(x) = \sqrt{x}(\tan(x) + \cos(x)) \). Find \( h'(x) \).

**Answer:** 
\[
\sqrt{x}(\sec^2(x) - \sin(x)) + \frac{1}{2}x^{-1/2}(\tan(x) + \cos(x))
\]

**Solution:** We write \( h(x) = f(x)g(x) \), where \( f(x) = \sqrt{x} \) and \( g(x) = \tan(x) + \cos(x) \). The product rule then says that \( h'(x) = f(x)g'(x) + f'(x)g(x) \). This gives:

\[
h'(x) = \sqrt{x}(\sec^2(x) - \sin(x)) + \frac{1}{2}x^{-1/2}(\tan(x) + \cos(x)).
\]

**Marking scheme:** 2 marks for correct answer with good work. 1 part mark available for:
- correct statement or application of product rule (with an error in a derivative somewhere, etc.)
- correct derivatives of BOTH trig functions

(b) The position of a particle at time \( t \) is given by \( s(t) = \frac{1}{3}t^3 - \frac{5}{2}t^2 + 4t \). Find all values of \( t \) where the particle has zero velocity.

**Answer:** \( t = 1, t = 4 \)

**Solution:** Given a position function, a formula for instantaneous velocity is given by 

\[
v(t) = s'(t) = t^2 - 5t + 4.
\]

So we need to solve the equation \( t^2 - 5t + 4 = 0 \). We factor the quadratic to get \( (t - 1)(t - 4) = 0 \), and so the particle has zero velocity at \( t = 1 \) and \( t = 4 \).
(Using the quadratic formula also works.)

**Marking scheme:**
- 1 mark for derivative of \( s(t) \)
- 1 mark for completely solving \( v(t) = 0 \)
Long answer question — you must show your work

3. \textbf{4 marks} Let \(f(x)\) be a function such that \(f(e) = 2\) and \(f'(e) = -3\). Let \(g(x) = x \cdot f(e^x)\). Find the equation of the line tangent to \(y = g(x)\) at \(x = 1\).

\textbf{Solution:} The equation of the line tangent to \(y = g(x)\) at \(x = 1\) is

\[ y = g'(1)(x - 1) + g(1). \]

We have \(g(1) = 1 \cdot f(e^1) = 1 \cdot 2 = 2\).

Now we compute \(g'(x)\). By the product rule,

\[ g'(x) = x \cdot \frac{d}{dx}[f(e^x)] + f(e^x) \cdot 1. \]

By the chain rule,

\[ \frac{d}{dx}[f(e^x)] = f'(e^x) \cdot e^x. \]

Therefore \(g'(x) = x \cdot f'(e^x) \cdot e^x + f(e^x)\), and so \(g'(1) = f'(e) \cdot e + f(e) = -3e + 2\).

Substituting these values into the equation of the tangent line, we obtain

\[ y = (-3e + 2)(x - 1) + 2. \]

\textbf{Marking scheme:}

- 1 mark general equation of tangent line (explicitly or implicitly, i.e. if the answer is correct but the general form is never stated, that’s okay)
- 1 mark for application of the product rule in computing \(g'(x)\)
- 1 mark for application of chain rule in computing \(\frac{d}{dx}[f(e^x)]\)
- 1 mark for the correct answer
- If student correctly finds equation of tangent line to \(y = f(x)\) at \(x = e\) (instead of \(y = g(x)\) at \(x = 1\)): at most 1 mark
- If student evaluates \(\frac{d}{dx}[f(e^x)]\) as just \(f'(e^x)\), but everything else is perfect: 3 marks (missing only the chain rule mark)