MECH 221 MATH LEARNING GUIDE — WEEK ELEVEN (starts 2014-12-01)

Lecture Schedule.

2014-12-01 (Mon): MATH 27, Linear Approximations
2014-12-03 (Wed): MATH 28, Taylor Polynomials and Applications

Overview. This week we make a quick review of differential calculus, refreshing the concepts of linear and higher-order approximation.

Learning Goals. You have mastered this week's material when you can ...

1. (Point-based linear approximation in 1D) Given some function $f(x)$ and point of interest a, find the unique linear function $L(x)$ for which $L(a) = f(a)$ and $L'(a) = f'(a)$. Use the linear approximation $f(x) \approx L(x)$ for $x \approx a$ to solve problems.

2. (Data-driven linear approximation in 1D) Given numerical values $z_1 = f(x_1)$ and $z_2 = f(x_2)$ for points $x_1 \neq x_2$, find the unique linear function $L(x)$ for which $L(x_1) = f(x_1)$ and $L(x_2) = f(x_2)$. Use the linear approximation $f(x) \approx L(x)$ for x between x_1 and x_2 to solve problems.

3. (Point-based linear approximation in 2D) Given some function $f(x, y)$ and point of interest (a, b), find the unique linear function $L(x, y) = a_0 + a_1 x + b_1 y$ for which the following equations hold *at the point of interest,* (a, b):

$$L = f, \qquad \frac{\partial L}{\partial x} = \frac{\partial f}{\partial x}, \qquad \frac{\partial L}{\partial y} = \frac{\partial f}{\partial x}.$$

Use the resulting approximation $f(x, y) \approx L(x, y)$ for $(x, y) \approx (a, b)$ to solve problems.

4. (Data-driven approximation in 2D) Given numerical values for a general function $f(x, y)$ at the corners of a rectangle in the (x, y)-plane, use iterated linear interpolation to predict values at points inside that rectangle. (Treat only rectangles whose sides are parallel to the coordinate axes. *Note*: The outcome of three simple linear calculations is not actually a linear function of (x, y).)

5. (Point-based higher-order approximations in 1D) Explain how matching derivatives up to order N at the origin leads to the N-th order Taylor Polynomial approximation $P_N(t)$ for a given function $f(t)$.

6. Explicitly construct the Taylor polynomial $P_N(t)$ for a given function f and order N.

7. Recognize the Taylor series at 0 for the standard functions $\sin(x)$, $\cos(x)$, $\exp(x)$, $1/(1-x)$, and know the set of real numbers x for which they converge.

8. Derive new Taylor series at 0 by substitution and algebraic manipulation of known series.

9. Use an approximation based on Taylor series to approximate a given nonlinear ODE with a linear one. (Classroom example: $\ddot{y} + e^{9y} = 1$, for $|y| \ll 1$.)

10. Use the general form of a Taylor series around 0 for an arbitrary function f to derive approximate formulas for computing derivatives of various orders, including error information. Sample:

$$f'(0) = \frac{f(h) - f(-h)}{2h} + O(h^2).$$

Textbook Sections.

- ☐ **JL 7.1 — Power Series:** There is a little more here than we really need, but it's approachable. Covers goals 1, 5, 6, 7, 8. Read the whole section, and try exercises #7.1.6, 7.1.8–10.
- ☐ **Wetton's Online Notes — 3. Taylor Polynomial Approximation:** Simple direct approach to material shown in class, covers goals 5–7.

- ☐ **Wetton — 4. Interpolation** : Simple direct approach to material shown in class. Covers goals 2 and 4.
- ☐ **Wetton — 5. Differentiation:** Simple direct approach to material shown in class. A nice application of Taylor's Theorem. Covers goal 10.

Final Examination Preview. Two of the upcoming final examination sessions will include marks labelled "Math". The questions will sample all math topics from the whole course, with only a slight bias in favour of material that has not previously been tested during the term. All math topics may appear on either test.

- ☐ Monday 15 Dec, 12:00-15:00: Electrical+Math test, with 2 short math problems (5 marks each), 1 long math problem (25 marks), and 1 combined Math/EE problem (12.5 marks for each subject). [For "Electrical" marks, expect another 3 short problems and 2 long problems, in addition to the EE/Math combination.]
- ☐ Tuesday 16 Dec, 12:00-15:00: Dynamics+Math test, with 2 short math problems (5 marks each), 1 long math problem (25 marks), and 1 combined Math/Dynamics problem (12.5 marks for each subject). [For "Dynamics" marks, expect another 3 short problems and 2 long problems, in addition to the Dynamics/Math combination.]