MECH 221 MATH LEARNING GUIDE — WEEK NINE (starts 2014-11-17)

Lecture Schedule.

2014-11-17 (Mon):	MATH 22, Shift Theorem #2 Examples; Dirac's Delta Function
2014-11-19 (Wed):	MATH 23, Impulses in ODE's and Physics; Review of Plane ODE systems

Overview. This week we continue our work with the Laplace Transform, focussing on Shift Theorem #2 and applications of the Dirac δ-"function".

Learning Goals. You have mastered this week's material when you can ...

1. Convert a given piecewise specification of a right-continuous function into a sum of terms involving the Heaviside step function.
2. Convert a given sum of terms involving the Heaviside step function into a piecewise specification for the functioin involved.
3. Find the Laplace Transform of functions like $u(t-a)f(t-a)$, using
$$\mathcal{L}\left\{u(t-c)f(t-c)\right\} = e^{-ct}\mathcal{L}\left\{u(t)f(t)\right\}.$$
4. Find the inverse Laplace Transform of functions like $e^{-cs}F(s)$, usoing
$$\mathcal{L}^{-1}\left\{e^{-cs}F(s)\right\} = u(t-c)f(t-c); \qquad \text{here} \quad f(t) = \mathcal{L}^{-1}\left\{F(s)\right\}.$$
5. Find the Laplace Transform of any function with the form $g(t)\delta(t-c)$, where $c \geq 0$ is some real number and g is some function continuous at c.
6. Solve nonhomogeneous initial-value problems whose forcing functions involve jump discontinuities, piecewise-defined functions, and/or impulses.
7. Set up and solve a differential equation for a simple dynamical and electrical system with a discontinuous or impulsive input.
8. Given a constant-coefficient linear differential equation or system and a scalar output variable, derive formulas and draw qualitative conclusions for
   - ☐ the frequency response (input $\cos(\omega t)$, various $\omega > 0$),
   - ☐ the step response (Heaviside input $u(t)$, all-0 initial values),
   - ☐ the impulse response (Dirac input $\delta(t)$, all-0 initial values).
9. Recognize and correctly interpret the phrase "Laplace Transform of the Impulse Response" as a synonym for "Transfer Function".

Textbook Sections.

- ☐ **JL 6.3 — Convolution:** In class we showed that $\mathcal{L}\left\{f(t)g(t)\right\} \neq F(s)G(s)$; thus also $\mathcal{L}^{-1}\left\{F(s)G(s)\right\} \neq f(t)g(t)$. This makes it natural to ask if there is any other way, perhaps more sophisticated, to construct $\mathcal{L}^{-1}\left\{F(s)G(s)\right\}$ from $f(t)$ and $g(t)$. The answer is, "yes, convolution," but we didn't have time to investigate this topic in class. It will not be examined.
- ☐ **JL 6.4 — Dirac Delta and Impulse Response:** Read this whole section. Solve 6.4.1–3, 6.4.6, 6.4.8, 6.4.101–102, 6.4.104–105.
- ☐ **WFT 8.5 — Constant Coefficient Equations with Piecewise Continuous Forcing Functions:** Try exercises #1–20, 21, 22.
- ☐ **WFT 8.6 — Convolution:** This is useful stuff, but we ran out of time for it. Skip it.
- ☐ **WFT 8.7 — Constant Coefficient Equations with Impulses:** Try #1–25, 29, 30.
- ☐ **WFT 8.8 — A Brief Table of Laplace Transforms:** Spend some time looking at this table. Notice the places where the textbook's chosen examples differ from ours. Decide if this table contains entries that you want to copy onto your personal formula card; conversely, notice which lines of the table in the Mech 2 formula book are not given here in the text.

Next Week's Test. On Thursday 27 November 2014, there will be a 110-minute test starting at 08:00. Out of the 75 marks available in total, the number designated for "Math" is 0.