

**MECH 221 MATH LEARNING GUIDE — WEEK THREE (starts 2014-10-06)**  
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**Lecture Schedule.**

- 2014-10-06 (Mon): MATH 06, Complex Numbers for ODE's  
2014-10-08 (Wed): MATH 07, Exponential Shift, II

**Learning Goals.** The first few goals are held over from last week. You have mastered this week's material when you can ...

1. Given a complex number or expression  $z = a + ib$  ( $a, b \in \mathbb{R}$ ), find  $\bar{z}$ ,  $\Re(z)$ ,  $\Im(z)$ , and  $|z|$ .
2. Given two complex numbers or complex-valued expressions  $z_1 = a_1 + ib_1$  and  $z_2 = a_2 + ib_2$ , express each of the quantities  $z_1 + z_2$ ,  $z_1 - z_2$ ,  $z_1 z_2$ ,  $z_1/z_2$  in the standard form for  $a + ib$  ( $a, b \in \mathbb{R}$ ).
3. Recall Euler's Formula,  $e^{i\theta} = \cos(\theta) + i \sin(\theta)$ , and use it to create new identities.
4. Determine the angular frequency  $\omega$ , given the graph of  $y = \cos(\omega t + \phi)$ .
5. Given a complex number or expression  $z$ , find all  $R > 0$  and  $\phi \in \mathbb{R}$  satisfying

$$z = Re^{i\phi}.$$

6. Recognize the equivalent harmonic-motion forms named AB, C, and D on the official formula sheet. Convert from any one form to any other.
7. Write the real-valued general solution of  $L[y] = 0$  for any linear differential operator  $L$  of order 1 or 2, assuming the coefficients are real constants. (Handle correctly each of 3 scenarios for the characteristic polynomial  $p(s)$ : distinct real roots, repeated real roots, or a pair of complex roots.)
8. Express any function of type PET (poly $\times$ exp $\times$ trig) as the real part of a function of type PE (poly $\times$ exp).
9. Find a particular solution for any ODE of the form  $L[y] = f$  where the constant-coefficient operator  $L$  has real coefficients,  $f$  has type PET, and  $n \leq 2$ .
10. Solve initial-value problems involving ODE's of the type described in goal 9.

**Textbook Sections.** This week we use complex numbers to add value to the knowledge gained so far.

- JL 2.2 — Constant coefficient second order linear ODEs:** Read this whole section. Pages 53–55 deal with complex characteristic roots. Practice on problems #2.2.10–11, 2.2.103, 2.2.105.
- JL 2.3 — Higher order linear ODEs:** Practice on problems #2.3.102–103.
- JL 2.5 — Nonhomogeneous equations:** Examples showing some simple sinusoidal inputs are shown on pages 71–72. Try problems #2.5.4, 2.5.6, 2.5.101, 2.5.105.
- WFT 5.2 — Constant Coefficient Homogeneous Equations:** Look at the answers for problems #1–17, 22–28, 34. Find the ones involving sinusoids, and attempt those questions.
- WFT 5.3 — Nonhomogeneous Linear Equations:** Here is the general theory for handling nonhomogeneous equations even when the coefficients in the operator  $L$  are time-varying. Think about what it tells us in the case we prioritize right now, where all coefficients are constant. Use familiar methods to solve #1–4, 7, 16–23, 33–38.
- WFT 5.5 — The Method of Undetermined Coefficients, II:** Here is a detailed discussion of  $L[y] = f$  with  $f$  of type PET. On pages 230–231, Trench uses exponential

shift to strip off only the real exponential part of  $f$ , leaving the sinusoid to be handled by real methods. This is an interesting and valid alternative to the fully complexified approach advocated in class. Practice, by either method, on #1–17, 22–26, 27–32, 40.

**Next Week's Test.** On Thursday 16 October 2014, there will be a 110-minute test starting at 08:00. Out of the 75 marks available in total, the number designated for “Math” is 0.