

MECH 221 MATH LEARNING GUIDE — WEEK TWO (starts 2014-09-29)

UBC MECH 2 Materials ©2014 by Philip D. Loewen

Lecture Schedule.

- 2014-09-29 (Mon): MATH 03, Exponential Shift, I
2014-10-01 (Wed): MATH 04, Complementary Solutions
2014-10-03 (Fri): MATH 05, Complex Numbers

Learning Goals. You have mastered this week's material when you can ...

1. Apply the exponential shift formula with a constant-coefficient linear differential operator

$$L[y] = a_n y^{(n)} + a_{n-1} y^{(n-1)} + \dots + a_2 \ddot{y} + a_1 \dot{y} + a_0 y$$

to simplify $L[e^{kt}u(t)]$. Recall: with the characteristic polynomial p ,

$$L[e^{kt}u(t)] = e^{kt} \left(\frac{p^{(n)}(k)}{n!} u^{(n)}(t) + \dots + \frac{p''(k)}{2!} \ddot{u}(t) + \frac{p'(k)}{1!} \dot{u}(t) + p(k)u(t) \right).$$

2. Use splitting and superposition to find *a particular solution* for any ODE of the form $L[u] = f(t)$, where $f(t)$ is a sum of several terms, each having the form $q(t)e^{kt}$ for some polynomial $q(t)$ and some real constant k .
3. Find *the general solution* for ODE's of the form $L[y] = f(t)$, where $f(t)$ is a sum of polynomial×exponential pieces as mentioned in Goal 3, and all roots of the characteristic polynomial $p(s)$ are real.
4. Solve *initial-value problems* for ODE's of the type described in Goal 4.
5. Recognize the words and phrases printed in italics above, and be able to solve problems in which the instructions are presented in prose using those terms.
6. Given a complex number or expression $z = a + ib$ ($a, b \in \mathbb{R}$), find \bar{z} , $\Re(z)$, $\Im(z)$, and $|z|$.
7. Given two complex numbers or complex-valued expressions $z_1 = a_1 + ib_1$ and $z_2 = a_2 + ib_2$, express each of the quantities $z_1 + z_2$, $z_1 - z_2$, $z_1 z_2$, z_1/z_2 in the standard form for $a + ib$ ($a, b \in \mathbb{R}$).
8. Recall Euler's Formula, $e^{i\theta} = \cos(\theta) + i \sin(\theta)$, and use it to create new identities.
9. Given a complex number or expression z , find all $R > 0$ and $\phi \in \mathbb{R}$ satisfying

$$z = Re^{i\phi}.$$

10. Recognize the equivalent harmonic-motion forms named AB, C, and D on the official formula sheet. Convert from any one form to any other.

Textbook Sections. This week we make the transition from finding just one particular solution to making a complete catalogue of all solutions. Our textbooks present the ideas involved before the story about particular solutions, so it's now possible—and recommended!—to read through the texts in linear order.

- **JL 2.1 — Second order linear ODEs:** Read this whole section. Try problems #2.1.1, 2.1.2–4, 2.1.103.
- **JL 2.2 — Constant coefficient second order linear ODEs:** Read this whole section. Pages 51–52 deal with the case of real characteristic roots, which will be our focus for this week. But by Friday, complex numbers will start to appear in class, so the material on Euler's formula and how it helps us will be worth knowing soon enough. Practice on problems #2.2.1–2.2.5 (embedded in the text), 2.2.6–2.2.9, 2.2.10–2.2.13, 2.2.101–2.2.105.

- **JL 2.3 — Higher order linear ODEs:** Practice on problems #2.3.1–2.3.6, 2.3.101–2.3.103.
- **JL 2.4 — Mechanical vibrations:** This is important enough to deserve a good treatment. Skip it for now. We'll come back to it next week.
- **JL 2.5 — Nonhomogeneous equations:** We have done half of this now. More next week. The practice problems recommended in Week One are still the only ones to master.
- **WFT 5.0 — Preamble:** Read this partial page.
- **WFT 5.1 — Homogeneous Linear Equations:** The general theory shown here reveals that many of the general properties we are seeing in class for constant-coefficient equations apply also when the coefficients depend on t . Our constant-coefficient assumption will remain in force for some time yet, so focus your thinking on what these general statements have to offer in our special case. Practice on problems #1–3, 38–40.
- **WFT 5.2 — Constant Coefficient Homogeneous Equations:** Read the whole section, then try problems #1–17, 22–28, 34.
- **WFT 5.3 — Nonhomogeneous Linear Equations:** Here is the general theory for handling nonhomogeneous equations even when the coefficients in the operator L are time-varying. Think about what it tells us in the case we prioritize right now, where all coefficients are constant. Use familiar methods to solve #1–4, 7, 16–23, 33–38.
- **WFT 5.4 — The Method of Undetermined Coefficients, I:** The story about finding particular solutions is familiar for us (see Week One), but now we can practice finding the general solution as well. Try #15–19 and 20–23.

Next Week's Test. On Friday 10 October 2014, there will be a 50-minute test starting at 08:00. The math questions will be based on topics listed on the Learning Guides for Weeks One and Two.