Math 403 Problem Set 5

Due in class on Wednesday 3 March 2010

The general approach used to analyze extremals for the rocket car in class also works for the problems below. Use it to show that every extremal control is piecewise constant with value either -1 *or* $+1$*, and that the switches in control value are controlled by the sign of some function you can capture quite explicitly. Understanding the system paths in the* (x_1, x_2)*-plane for each interval where the control is constant is a good way to start piecing together the solution. The differential equations are simple enough to solve by hand, and explicit algebraic solutions will be the key to success. But when it comes to sketching the requested attainable sets, you are welcome to enlist the help of some computer package.*

1. Let $\mathcal{A}(T)$ denote the set of all vectors $x(T)$ corresponding to solutions for this system:

$$\begin{aligned} \dot{x}_1(t) &= \quad x_2(t), & x_1(0) &= 0, \\ \dot{x}_2(t) &= -x_1(t) \quad + u(t), & x_2(0) &= 0, \\ |u(t)| &\le 1. \end{aligned}$$

(a) Find and sketch $\mathcal{A}(\pi/2)$.

(b) Find and sketch $\mathcal{A}(\pi)$.

(c) Find and sketch $\mathcal{A}(3\pi/2)$.

2. Given the control system

$$\begin{aligned} \dot{x}_1 &= -x_1 - u \\ \dot{x}_2 &= -2x_2 - 2u, \qquad u \in [-1, 1], \end{aligned}$$

find the attainable set $\mathcal{A}(T; 0, U)$ for every $T > 0$. Show that every such set is contained in $\mathbb{B}[0; \sqrt{2}]$, and that $\mathcal{A}(T; 0, U)$ approaches a definite limit as $T \to \infty$. Contrast these properties with those of the attainable set for the rocket car discussed in class. Then identify the different characteristics of the respective system matrices A that account for these different behaviours.