

Standard Eigenfunction Series—Example

UBC M316 Resources by Philip D. Loewen

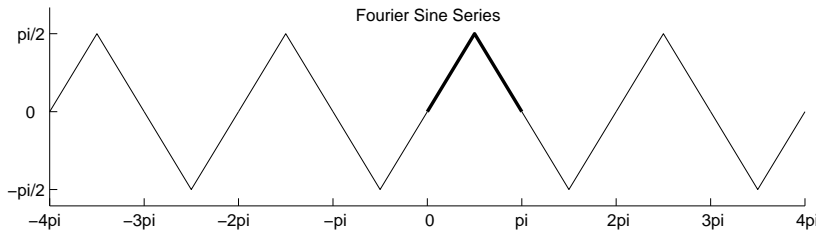
Here are four different expansions for the same function on the interval $0 < x < \pi$:

$$f(x) = \begin{cases} x, & \text{if } 0 < x \leq \pi/2, \\ \pi - x, & \text{if } \pi/2 < x < \pi. \end{cases}$$

All four match the f exactly on the basic interval (drawn in bold), but have different behaviour outside the basic interval.

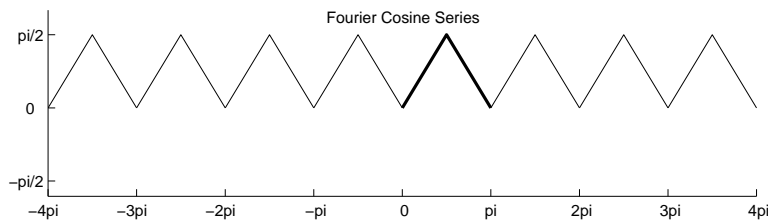
Fourier Sine Series. $f(x) = \tilde{f}(x) = \sum_{n=1}^{\infty} b_n \sin(nx)$ for $0 < x < \pi$, where

$$b_n = \frac{4}{\pi n^2} \sin\left(\frac{n\pi}{2}\right).$$



Fourier Cosine Series. $f(x) = \frac{1}{2}a_0 + \sum_{n=1}^{\infty} a_n \cos(nx)$ for $0 < x < \pi$, where

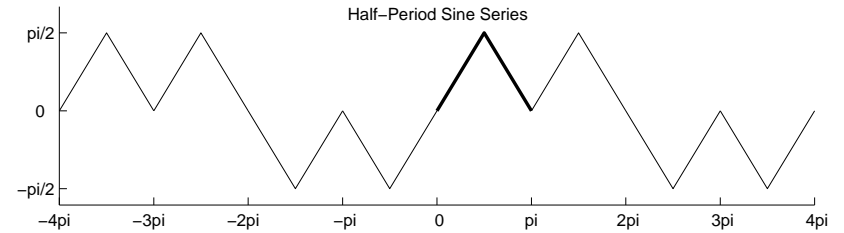
$$a_n = \frac{2}{\pi n^2} \left[2 \cos\left(\frac{n\pi}{2}\right) - 1 - (-1)^n \right], \quad n \geq 1; \quad a_0 = \frac{\pi}{2}.$$



Half-Period Sine Series. $f(x) = \tilde{f}(x) = \sum_{n=1}^{\infty} p_n \sin\left(\frac{2n-1}{2}x\right)$ for $0 < x < \pi$,

where

$$p_n = \frac{8}{\pi(2n-1)^2} \left[\sqrt{2} \sin\left(\frac{n\pi}{2}\right) - \sqrt{2} \cos\left(\frac{n\pi}{2}\right) + (-1)^n \right].$$



Half-Period Cosine Series. $f(x) = \tilde{f}(x) = \sum_{n=1}^{\infty} q_n \cos\left(\frac{2n-1}{2}x\right)$ for $0 < x < \pi$,

where

$$q_n = \frac{8}{\pi(2n-1)^2} \left[\sqrt{2} \sin\left(\frac{n\pi}{2}\right) + \sqrt{2} \cos\left(\frac{n\pi}{2}\right) - 1 \right].$$

