UBC Mathematics 257/316—Assignment 2
Due in class on Tuesday 27 May 2014

Themes: Power series solutions around ordinary points, constant-coefficient ODE's, homogeneous ODE's of Euler type.

1. Find the recurrence equation for the coefficients of a series solution of

$$y'' - xy' + 3y = 0, \quad y(0) = 1, \quad y'(0) = 0,$$

and the first four nonzero terms. What is the radius of convergence of the series?

2. Solve these differential equations using a power series about the given point x_0. In each case, (i) find the recurrence relation, (ii) write the first four terms in each of two linearly independent solutions (unless the series terminates sooner), and (iii) for each series solution, find an open interval of $\mathbb{R}$ in which the differential equation is satisfied.

(a) $(1 + x^2)y'' - 4xy' + 6y = 0$, $x_0 = 0$.

(b) $2y'' + (1 + x)y' + 3y = 0$, $x_0 = 2$.

3. Use the substitution $t = x - 2$ to find two linearly independent series solutions of

$$y'' + (x - 2)^2 y' + (x^2 - 4)y = 0$$

in powers of $(x - 2)$. State an open interval in which both series are guaranteed to solve the equation.

4. Review second-order linear differential equations with constant coefficients and their applications. This is prerequisite material for the current course. Our official course textbook by William F. Trench covers all you need (and more) in Chapters 5–6.

(a) Find the general solution: $y'' + 4y' + 8y = 0$.

(b) Suppose the function $y(x)$ satisfies the differential equation in part (a), and we also have $y(1) = 0$ with $y'(1) \neq 0$. List all the zeros of y. That is, find all real numbers x such that $y(x) = 0$.
Hint: The amplitude-phase form of the general solution is convenient here. See pages 270–271 and 279–280 of the Trench text mentioned earlier.

(c) Determine the set of all real constants c for which the following statement is correct: *One has $y(x) \to 0$ as $x \to \infty$ for every function y satisfying*

$$y'' + 2cy' + 8y = 0.$$

(d) Find the constant v_0, given the following facts:

$$\left[\quad y'' - 4y = 0, \quad y(0) = \sqrt{2},\ y'(0) = v_0 \quad\right] \quad\Longrightarrow\quad \lim_{x\to\infty} y(x) = 0.$$

(Continued . . .)

5. Solve for $y(t)$: $y' + 2t^{-1}y = t^{-2}e^t$, $t > 0$; $y(1) = 0$.

6. (a) Find the general solution: $x^2y'' + 3xy' + 5y = 0$, $x > 0$.

(b) Find the general solution: $(x-1)^2y'' - 5(x-1)y' + 9y = 0$, $x > 1$.

(c) Solve: $x^2y'' + 8xy' + 12y = 0$, $x > 0$, $y(1) = 4$, $y'(1) = -3$.

(d) Find all values of α for which all solutions of $y'' + \alpha x^{-2}y = 0$ approach 0 as $x \to 0^+$.