1. Suppose you are walking on the hill which satisfies the equation \( z = x^2 + xy + 2y^2 \):
   (a) at the point (1, 1) along which direction you are rising the quickest? And along this direction what is your slope? (6 points)
   
   \( \nabla f = \langle 2x + y, x + 4y \rangle \)
   
   at (1, 1), \( \nabla f = \langle 3, 5 \rangle \) along the direction of
   
   \[ \overrightarrow{u} = \frac{\nabla f}{||\nabla f||} = \left\langle \frac{3}{\sqrt{34}}, \frac{5}{\sqrt{34}} \right\rangle \]
   
   is rising the quickest.

   \[ \text{slope} = D_{\overrightarrow{u}} f = ||\nabla f|| = \sqrt{34}. \]

   (b) on the unit disk \( x^2 + y^2 \leq 1 \) find the highest and lowest points on the hill. (4 points)

   \( \nabla f = \langle 2x + y, x + 4y \rangle = \langle 0, 0 \rangle. \)

   Critical point (0, 0), \( z(0,0) = 0. \) — Lowest point.

   On the boundary take \( x = \cos \theta, y = \sin \theta. \)

   \[ z(\theta) = z(x,y) = \cos^2 \theta + \cos \theta \sin \theta + 2 \sin^2 \theta \]

   \[ = 1 + \cos \theta \sin \theta + \sin^2 \theta = 1 + \frac{\sin 2\theta}{2} + \frac{1 - \cos 2\theta}{2} \]

   \[ z'(\theta) = \cos 2\theta + \sin 2\theta = 0 \]

   \[ \Rightarrow \tan 2\theta = -1, \quad 2\theta = \frac{3\pi}{4} \text{ or } \frac{7\pi}{4}, \quad \theta = \frac{3\pi}{8} \text{ or } \frac{7\pi}{8} \]

   For these two points: \( \cos \left( \frac{3\pi}{8} \right), \sin \left( \frac{3\pi}{8} \right) \), \( z = 1 + \frac{1}{2} \cdot \frac{\sqrt{2}}{2} + \frac{1 - \frac{\sqrt{2}}{2}}{2} = \frac{3}{2} + \frac{\sqrt{2}}{2} \) — Higher point.

   \( \cos \left( \frac{7\pi}{8} \right), \sin \left( \frac{7\pi}{8} \right) \), \( z = 1 - \frac{1}{2} \cdot \frac{\sqrt{2}}{2} + \frac{1 - \frac{\sqrt{2}}{2}}{2} = \frac{3}{2} - \frac{\sqrt{2}}{2} \) — So, higher point.