SOLUTIONS

Math 200-108 Quiz 5 (10 points)

NAME: 
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NOTE: WRITE YOUR ANSWERS DIRECTLY AND YOU DON'T NEED TO EXPLAIN!!!

1. Write the integral \( I = \iiint_D f(x, y, z) \, dx \, dy \, dz \) in \( dz \, dy \, dx \) and \( dx \, dy \, dz \) order, where \( D \) is bounded by \( z = 1 - x^2 \), \( z = 1 - y^2 \) in the first octant. (7 points)

\[
\iiint_D f(x, y, z) \, dz \, dy \, dx = \left( \int_0^1 \int_0^{1-x^2} \int_0^{1-y^2} + \int_0^1 \int_0^{x^2} \int_0^{1-y^2} \right) f(x, y, z) \, dz \, dy \, dx.
\]

\( dx \, dy \, dz \): projection of \( y - z \) is bounded by \( z = 1 - y^2 \)

\[
\iiint_D f(x, y, z) \, dx \, dy \, dz = \int_0^1 \int_0^{\sqrt{1-z}} \int_0^{\sqrt{1-z}} f(x, y, z) \, dx \, dy \, dz.
\]

-1 if missing integrand.

2. Write the integral \( I = \iiint_D (x + y + z) \, dV \) with cylindrical coordinates, where \( D \) is in the first octant and under the paraboloid \( z = 4 - x^2 - y^2 \). You don't need to compute this integral!!! (3 points)

\[
I = \int_0^\frac{\pi}{2} \int_0^2 \int_0^{4-r^2} (r \cos \theta + r \sin \theta + z) \, r \, dz \, dr \, d\theta.
\]

-1 if integrand wrong, \( z \) in terms of \( x, y, z \).