200-108 Quiz 3 (10 points)

NAME: ____________________

UBC ID: ____________________

1. Given a function \( z = f(x, y) = 2x^2 + y^2 + 2x \),
   (a) At the point \((0, 1)\), along which direction (unit vector) the function is fastest increasing? Find the slope of this increasing along this direction. (3 points)

   \[ \nabla f = \langle 4x + 2, 2y \rangle. \]

   at \((0, 1)\), \( \nabla f = \langle 2, 2 \rangle \). so fastest increasing direction is same to \( \nabla f \): \( \vec{u} = \frac{\langle 2, 2 \rangle}{\sqrt{2^2 + 2^2}} = \langle \frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}} \rangle \)

   slope is \( D_{\vec{u}} f = ||\nabla f|| = 2\sqrt{2}. \)

   (b) Find the absolute maximum and minimum on the unit disk \( \{x^2 + y^2 \leq 1\} \). (3 points)

   \[ \left\{ \begin{array}{l} f_x = 4x + 2 = 0 \text{ critical point. } (-\frac{1}{2}, 0), \vspace{5mm} \\
   f_y = 2y = 0 \end{array} \right. \]

   \[ f(-\frac{1}{2}, 0) = -\frac{1}{2}. \]

   on the boundary \( x^2 + y^2 = 1 \), take \((x, y) = (\cos \theta, \sin \theta)\).

   \[ f(x, y) = 2\cos^2 \theta + \sin^2 \theta + 2\cos \theta = \cos^2 \theta + 2\cos \theta + 1 = (1 + \cos \theta)^2 \] so two extreme values: \( \theta = 0 \) and \( \pi \).

   \[ f(\pm 1, 0): \quad f(1, 0) = 4, \quad f(-1, 0) = 0 \]

   (or \( \int f_{\theta} = 2(1 + \cos \theta)(-\sin \theta) = 0 \Rightarrow \theta = 0, \pi \))

   Absolute max: \( f(1, 0) = 4 \). \quad \text{min: } f(-\frac{1}{2}, 0) = -\frac{1}{2}. \)

2. Given a contour graph as follows, find out all the points satisfying the following requirements: (You can get point for each one only if you get a totally correct answer) (4 points)
(a) Local maximum: $C\ E$
(b) Local minimum: $A$
(c) Saddle points: $B\ D$
(d) The directional derivative along positive x-axis is positive: $F\ H$