Math 538: Algebraic Number Theory
Spring Term, 2024

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Course Website	`http://www.math.ubc.ca/~lior/teaching/2324/538_W24/`
Contact me at	`lior@math.ubc.ca` or at MATX 1112
My Website	`http://www.math.ubc.ca/~lior/`
Class	WF 9:30-11:00 at ESB 4127 and on Zoom
Office Hours	WF 11:00-11:30 at ESB 4127 and on Zoom; evening office hours TBA
Textbook	None required; see below for recommendations
(Informal) Prerequisites	Introductory Algebra, Elementary Number Theory, Galois Theory

About the course

This will be a standard graduate course in Algebraic Number Theory.

Topics will include:

- Number fields, rings of integers, ideals and unique factorization. Finiteness of the class group.

- Valuations and completions; local fields.

- Ramification theory, the different and discriminant.

- Geometry of numbers: Dirichlet's Unit Theorem and discriminant bounds.

- Other topics if time permits

The main pre-requisites are basic algebra (rings and fields, rings of polynomials, unique factorization in Euclidean domains), basic number theory (modular arithmetic, factorization into primes) and Galois Theory, but no specific courses are required.

Textbook

There are many books titled "Algebraic Number Theory"; you can use any for your own reference. I personally like Lang [2] but if you plan to buy one book make it Neukirsch [3] which is more comprehensive and has exercises. Both of these books are available for free download through your university library's subscription to SpringerLink.

Evaluation and grading; levels of participation

The grade will be based on up to six eight problem sets. There will be no final exam. Students should note that each problem set is *comprehensive* and will contain many problems: more than what a reasonable student might do, so each participant will need to manage their workload: there will be different expectations from a student planning to work in the field relative to a student outside the field taking the course for interest.

Remote participation

The course will be simultaneously broadcast on Zoom and open for remote participation, including as a PIMS network-wide course.

Registration

I would appreciate every student interested in the course to send me an email describing their number theory experience and intenteded level of participation (see above).

1. Students at Canadian PIMS member universities may apply for graduate credit via the Western Deans' Agreement. Please be advised, in some cases students must enroll 6 weeks in advance of the term start date and will typically be required to pay ancillary fees to the host institution (as much as $270) or explicitly request exemptions. Please follow the link above for details of fees at specific sites.

2. Students at universities not covered by the WDA but which are part of the Canadian Association for Graduate Studies may still be eligible to register for this course under the Canadian University Graduate Transfer Agreement. Details of this program vary by university and are also typically subject to ancillary fees. Please contact your local Graduate Student Advisor for more information.

3. As an alternative to formal registeration remote students may enrol in a local "reading course" at their home university. In that case please have the faculty member responsible for the reading course (usually your advisor) write to me to request that I grade your homework and send a grade back.

4. Anyone (anywhere in the world) who is interested in mathematics and would like to learn some Number Theory is welcome to attend the lectures as well as the online office hours. To be clear I will only mark the homework of registered participants.

Online classes

1. All remote and in-person participants are strongly encourage to **interrupt the lectures** repeatedly with their questions (even "can you explain that again?" or "what do you mean by 'continuous'?"). Such questions are welcome, encouraged, and are **absolutely essential** for learning to take place in a course of this type.

2. All classes will be recorded, with the recordings placed on a public website (possibly YouTube or the PIMS MathTube). Participants who do not wish to be recorded may join the Zoom session under pseudonyms, and only ask questions via chat.

3. All course information will be posted on the course website and will remain accessible in perpetuity to anyone anywhere.

References

[1]   Borevich–Shafarevich, *Algebraic Number Theory*

[2]   Lang, *Algebraic Number Theory*

[3]   Neukirsch, *Algebraic Number Theory*