Math 100C – WORKSHEET 1
EXPRESSIONS AND ASYMPTOTICS

1. Asymptotics: simple expressions

(1) ⋆ Classify the following functions into *power laws / power functions* and *exponentials*: x^3, πx^{102}, e^{2x}, $c\sqrt{x}$, $-\frac{8}{x}$, 7^x, $8 \cdot 2^x$, $-\frac{1}{\sqrt{3}} \cdot \frac{1}{2^x}$, $\frac{9}{x^{7/2}}$, x^e, π^x, $\frac{A}{x^b}$.

(2) ⋆ How does the each expression behave when x is large? small? what is x is large but negative? ⋆⋆ Sketch a plot

(a) $1 - x^2 + x^4$ ("Mexican hat potential")

(b) $x^3 - x^5$

(c) $e^x - x^4$

(d) Wages in some country grow at 2% a year (so the wage of a typical worker has the form $A \cdot (1.02)^t$ where t is measured in years and A is the wage today). The cost of healthcare grows at 4% a year (so the healthcare costs of a typical worker have the form $B \cdot (1.04)^t$ where B is the cost today). Suppose that today's workers can afford their healthcare (A is much bigger than B). Will that be always true? Why or why not?

(e) Three strains of a contagion are spreading in a population, spreading at rates 1.05, 1.1, and 0.98 respectively. The total number of cases at time t behaves like

$$A \cdot 1.05^t + B \cdot 1.1^t + C \cdot 0.98^t \, .$$

(A, B, C are constants). Which strain dominates eventually? What would the number of infected people look like?

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(3) The (attractive) interaction between two hadrons (say protons) due to the strong nuclear force can be modeled by the *Yukawa potential* $V_{\mathrm{Y}}(r) = -g^2 \frac{e^{-\alpha m r}}{r}$ where r is the separation between the particles, and g, α, m are positive constants. The elecctrical repulsion between two protons is described by the Columb potential $V_{\mathrm{C}}(r) = kq^2 \frac{1}{r}$ where k, q are also positive constants. Which interaction will dominate for large distances? Will the net interaction be attractive or repulsive? Note that g^2 is much larger than kq^2.

2. Asymptotics of complicated expressions

(4) Describe the following expressions in words

(a) $e^{|x-5|^3}$

(b) $\frac{1+x}{1+2x-x^2}$

(c) $\frac{e^x + A \sin x}{e^x - x^2}$

(d) $\left(\frac{t+\pi}{t-\pi}\right) \sin\left(\frac{t+\pi}{2}\right)$

(5) For each of the functions in (a),(b),(c),(d) determine its asymptotics as $x \to 0$ and as $x \to \infty$.

(a) $\star$

(b) $\star$

(c) $\star\star\star\star$

(d) $\star\star\star$