1. Converge or Diverge?

(1) Determine, with explanation, whether the following series converge or diverge.

(a) (Alternating harmonic series) \( \sum_{n=1}^{\infty} \frac{(-1)^n}{n} \).

(b) \[ 1 - \frac{1}{4} + \frac{1}{3} - \frac{1}{16} + \frac{1}{5} - \frac{1}{36} + \frac{1}{7} - \frac{1}{64} + \frac{1}{9} - \frac{1}{100} + \frac{1}{11} - \frac{1}{144} + \cdots \]

(c) (Final 2014) \( \sum_{n=1}^{\infty} \frac{n \cos(\pi n)}{2^n} \)

(d) (Final 2011) \( \sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{n^p} = 1 - \frac{1}{2^p} + \frac{1}{3^p} - \frac{1}{4^p} + \cdots \) (your answer will depend on \( p \))

(2) Power series

(a) (Final 2013, variant) Decide whether the series \( \sum_{n=1}^{\infty} \frac{(-1)^n}{\sqrt{n}} (x + 2)^n \) converges or diverges at \( x = -1 \) and at \( x = -3 \).

(b) Decide whether the series \( \sum_{n=1}^{\infty} nx^n \) converges or diverges at \( x = 1 \) and \( x = -1 \).
2. Error estimates

(3) (a) It is known that $1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \frac{1}{5} - \frac{1}{6} + \cdots = \log 2$. How many terms are needed for the error to be less than 0.01?

(b) It is known that $1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \frac{1}{9} - \frac{1}{11} + \cdots = \frac{\pi}{4}$. How many terms are needed for the error to be less than 0.001?

(4) (MacLaurin expansions)

(a) It is known that $e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!}$. How close is $\frac{1}{2} - \frac{1}{6} + \frac{1}{24}$ to $\frac{1}{e}$? How many terms are needed to approximate $\frac{1}{e}$ to within $\frac{1}{1000}$?

(b) The error function is (roughly) given by $\text{erf}(x) = \sum_{n=0}^{\infty} \frac{(-1)^n}{n!(2n+1)} x^{2n+1}$. How many terms are needed to approximate $\text{erf}(\frac{1}{10})$ to within $10^{-11}$?