Math 101 – WORKSHEET 25
THE INTEGRAL TEST

1. REVIEW OF IMPROPER INTEGRALS

(1) Show that $\int_2^{\infty} \frac{dx}{x}$ diverges.

(2) Show that $\int_2^{\infty} \frac{dx}{x^2 + 5}$ converges.

(3) Evaluate $\int_0^{\infty} xe^{-x} \, dx$.

2. APPLYING THE INTEGRAL TEST

(4) Decide if each series converges or diverges
   (a) $\sum_{n=1}^{\infty} \frac{1}{n^p}$ (your answer will depend on $p$!)
   
   (b) $\sum_{n=1}^{\infty} \frac{n}{e^n}$

   (c) (Final 2014) $\sum_{n=2}^{\infty} \frac{1}{n \log n}$ (your answer will depend on $p$!)

   (d) $\sum_{n=1}^{\infty} \frac{1}{n^2 + 1}$

(5) The integral $\int_2^{\infty} x^3 + \sin x \, dx$ diverges. Why can’t we use the integral test to conclude that $\sum_{n=2}^{\infty} \frac{n + \sin n}{1 + n^2}$ diverges as well?
(6) Consider the series $\sum_{n=1}^{\infty} \frac{1}{n^2}$
(a) Show that $\sum_{n=N+1}^{\infty} \frac{1}{n^2} \leq \frac{1}{N}$.

(b) How many terms do we need to include to get an answer accurate to $10^{-5}$?

(7) (The harmonic series)
(a) Show that $\sum_{n=1}^{N} \frac{1}{n} \geq \log(N + 1)$

(b) Show that $\sum_{n=1}^{N} \frac{1}{n} \leq (1 - \log 2) + \log(N + 1)$