1. IMPROPER AT INFINITY

(1) For which values of $p$ does $\int_1^{\infty} \frac{1}{x^p} \, dx$ converge? Diverge?

(2) (Final, 2010) Evaluate $\int_{-\infty}^{-1} e^{2x} \, dx$. Simplify your answer as much as possible.

(3) Find a constant $C$ such that $\int_{-\infty}^{+\infty} \frac{C \, dx}{1+x^2} = 1$.

(4) We study $\int_{-\infty}^{+\infty} x \, dx$.
   (a) Evaluate $\int_{-T}^{T} x \, dx$.
   (b) Evaluate $\lim_{T \to \infty} \int_{-T}^{T} x \, dx$.
   (c) Does the integral converge?

(5) (Final, 2009) For what values of $p$ does $\int_{e}^{\infty} \frac{dx}{x(\log x)^p}$ converge?
2. IMPROPER AT FINITE POINTS

(6) For which values of \( p \) does \( \int_0^1 \frac{dx}{x^p} \) converge?

(7) (Math 103 Final, 2013) Evaluate the integral if it exists, otherwise show that it doesn’t: \( I = \int_0^2 \frac{dx}{1-x^2} \).

3. COMPARISON OF INTEGRALS

(7) Decide which of the following integrals converge

(a) (103 Final, 2012) \( \int_1^\infty \frac{1+\sin x}{x^2} \, dx \).

(b) \( \int_1^\infty \frac{3-\cos x}{x} \, dx \).

(c) (Bell curve) \( \int_{-\infty}^{+\infty} e^{-x^2} \, dx \).

(d) \( \int_0^1 \frac{dx}{\sqrt{x^2+\sin x}} \).

(e) (hard) \( \int_0^1 \frac{dx}{x^2+x^4} \).

(f) (hard) \( \int_0^\infty \frac{1000}{e^x} \, dx \).