1. Partial fractions expansion

(1) Apply Method 2 to find $A, B, C$ such that
\[
\frac{6x^2 - 26x + 26}{x^2 - 6x^2 + 11x - 6} = \frac{6x^2 - 26x + 26}{(x - 1)(x - 2)(x - 3)} = \frac{A}{x - 1} + \frac{B}{x - 2} + \frac{C}{x - 3}
\]

Solution: We have
\[
\begin{align*}
\frac{6x^2 - 26x + 26}{(x - 1)(x - 2)(x - 3)} &\sim 6 \cdot 1^2 - 26 \cdot 1 + 26 = \frac{6}{x - 1} \\
\frac{6x^2 - 26x + 26}{(x - 1)(x - 2)(x - 3)} &\sim 6 \cdot 2^2 - 26 \cdot 2 + 26 = \frac{2}{x - 1} \\
\frac{6x^2 - 26x + 26}{(x - 1)(x - 2)(x - 3)} &\sim 6 \cdot 3^2 - 26 \cdot 3 + 26 = \frac{1}{x - 1}
\end{align*}
\]
so $A = 3$, $B = 2$, $C = 1$.

(2) Now consider \[
\frac{8x - 10}{4x^3 - 4x^2 + 5x} = \frac{8x - 10}{x(4x^2 - 4x + 5)} = \frac{A}{x} + \frac{Bx + C}{4x^2 - 4x + 5}
\]

(a) Find $A$ using method 2

Solution: We have \[
\frac{8x - 10}{x(4x^2 - 4x + 5)} \sim 0 - \frac{10}{x(5)} = -\frac{2}{x}
\]
so $A = -2$.

(b) Calculate \[
\frac{8x - 10}{x(4x^2 - 4x + 5)} - \frac{A}{x} = \frac{Bx + C}{4x^2 - 4x + 5}
\]
to find $B, C$.

Solution: We have
\[
\begin{align*}
\frac{8x - 10}{x(4x^2 - 4x + 5)} - \frac{(-2)}{x} &= 1 \left[ \frac{8x - 10}{4x^2 - 4x + 5} + 2 \right] \\
&= 1 \left[ \frac{8x - 10 + 2(4x^2 - 4x + 5)}{4x^2 - 4x + 5} \right] \\
&= 1 \left[ \frac{8x^2 + 8x - 8x - 10 + 10}{4x^2 - 4x + 5} \right] \\
&= \frac{8x^2}{x(4x^2 - 4x + 5)} \\
&= \frac{8x}{4x^2 - 4x + 5}
\end{align*}
\]
so that $B = 8$ and $C = 0$.

(3) Finally consider \[
\frac{x^2}{(x+2)(2x-3)}
\]
Can we have $A, B$ such that $x^2 = A(x + 2) + B(2x - 3)$?

Solution: No, because the degrees don’t match.
2. Approximate Integration

Let $f(x) = \sin(x^2)$. Estimate $\int_0^1 f(x) \, dx$ using the trapezoid rule, the midpoint rule, and Simpson’s rule, with $n = 4$ in all cases. You may leave your answers in calculator-ready form.