(1) (Area between curves) Find the area of the finite region bounded by the $y$-axis, the graph of $y = \arcsin(x)$ and the line $y = \frac{\pi}{2}$.

Solution: We draw a sketch first. Slicing vertically, requires evaluating

$$\int_{x=0}^{x=1} (1 - \arcsin x) \, dx$$

which is painful. Slicing horizontally instead, we have $0 \leq y \leq \frac{\pi}{2}$ and at each $y$ the length of the slice is $x = \sin y$ so instead we compute

$$\int_{y=0}^{y=\pi/2} \sin y \, dy = [\cos y]_{y=0}^{y=\pi/2} = 1.$$

(2) Solids of revolution
(a) The area between the $x$-axis, the curve $y = x^2$ and the line $x = 5$ is revolved about the $x$-axis. What is the volume of the resulting region?

Solution: The volume is $\int_{x=0}^{x=5} \pi y^2 \, dx = \int_{x=0}^{x=5} \pi x^4 \, dx = \pi \left[ \frac{x^5}{5} \right]_{x=0}^{x=5} = 5^4 \pi = 625 \pi$.

(b) (Final, 2014) Find the volume of the solid generated by rotating the finite region bounded by $y = \frac{1}{x}$ and $3x + 3y = 10$ about the $x$-axis. It will be useful to sketch the region first.

Solution: The intersection points are where $x + \frac{1}{x} = \frac{10}{3}$ that is where $x^2 - \frac{10}{3} x + 1 = 0$ that is where $x = \frac{10/3 \pm \sqrt{(10/3)^2 - 4}}{2} = \frac{10 \pm \sqrt{64}}{6} = \frac{5 \pm 4}{3} = \frac{1}{3}, 3$. The volume is
therefore
\[ \pi \int_{x=1/3}^{x=3} \left( \left( \frac{10}{3} - x \right)^2 - \left( \frac{1}{x} \right)^2 \right) \, dx = \pi \int_{x=1/3}^{x=3} \left( \frac{100}{9} - \frac{20}{3} x + x^2 - x^{-2} \right) \, dx \]
\[ = \pi \left[ \frac{100}{9} x - \frac{10}{3} x^2 + \frac{x^3}{3} + \frac{1}{x} \right]_{x=1/3}^{x=3} \]
\[ = \pi \left[ \left( 300 - 90 + 9 + \frac{1}{3} \right) - \left( \frac{100}{27} - \frac{10}{27} + \frac{1}{81} + 3 \right) \right] \]
\[ = \pi \left[ \frac{275}{81} \right] = \frac{275}{81} \cdot \pi. \]

(c) The area between the $y$-axis, the curve $y = x^2$ and the line $y = 4$ is rotated about the $y$-axis. What is the volume of the resulting region?

**Solution:** Slicing perpendicular to the $y$-axis, we need to evaluate
\[ \int_{y=0}^{y=4} \pi x^2 \, dy = \int_{y=0}^{y=4} \pi y \, dy = \frac{\pi}{2} \left[ y^2 \right]_{y=0}^{y=4} = 8\pi. \]