Math 101 – SOLUTIONS TO WORKSHEET 4
THE FUNDAMENTAL THEOREM OF CALCULUS

(1) (Differentiating integrals) Evaluate
(a) \( \frac{d}{dx} \int_{0}^{x} e^{t^2} \, dt \)

**Solution:** By the FTC this is \( e^{x^2} \).

(b) \( \frac{d}{dx} \int_{x}^{1} e^{t^2} \, dt \)

**Solution:** \( \int_{x}^{1} e^{t^2} \, dt = -\int_{1}^{x} e^{t^2} \, dt. \) Applying the FTC we get \( -e^{x^2} \).

(c) (Final 2009) \( \frac{d}{dx} \int_{x}^{c} \cos t \, dt \)

**Solution:** Fix \( c \), and let \( F(u) = \int_{c}^{u} \cos t \, dt. \) Then \( \int_{x}^{c} \cos t \, dt = \int_{c}^{x} \cos t \, dt - \int_{c}^{x} \cos t \, dt \)

so we need to compute \( \frac{d}{dx} \left( F(e^x) - F(x^2) \right) \).

By the chain rule this is \( F'(e^x)e^x - F'(x^2)(2x) = \sqrt{\cos(e^x)}e^x - 2x \sqrt{\cos(x^2)}. \)

(d) (Final 2014) Let \( f(x) = \int_{1}^{x} 100(t^2 - 3t + 2)e^{-t^2} \, dt. \) Find the interval(s) on which \( f \) is increasing.

**Solution:** By the FTC, \( f'(x) = 100(x^2 - 3x + 2)e^{-x^2} = 100(x - 2)(x - 1)e^{-x^2}, \) which is positive on \( (-\infty, 1) \cup (2, \infty) \).

(2) Evaluate using anti-derivatives
(a) (Final 2012) \( \int_{1}^{2} \frac{x^2 + 2}{x^2 + 4} \, dx = \)

**Solution:** \( \int_{1}^{2} \left( 1 + \frac{2}{x^2} \right) \, dx = \left[ x - \frac{2}{x} \right]_{x=1}^{x=2} = (2 - 1) - (1 - 2) = 2. \)

(b) (Final 2007) \( \int_{-1}^{0} (2x - e^x) \, dx = \)

**Solution:** \( F(x) = x^2 - e^x \) is an anti-derivative, so \( \int_{-1}^{0} (2x - e^x) \, dx = \left[ x^2 - e^x \right]_{x=-1}^{x=0} = 0 - e^0 - ((-1)^2 - e^{-1}) = -2 + \frac{1}{e}. \)

(c) \( \int_{3}^{10} (x^{5/2} + e^{2x}) \, dx = \)

**Solution:** An anti-derivative is \( \frac{2}{7}x^{7/2} + \frac{1}{2}e^{2x} \) so the answer is \( \left[ \frac{2}{7}x^{7/2} + \frac{1}{2}e^{2x} \right]_{x=3}^{x=10} = \frac{2}{7}10^{7/2} + \frac{1}{2}e^{20} - \frac{2}{7}3^{7/2} - \frac{1}{2}e^{6}. \)

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