19. Work II (17/2/2017)

Goals:

(1) Work: diffuse body.
(2) Quiz

Last time: Work on pointlike object.

(1) Work done by applying constant force F along distance Δx is $W = F\Delta x$.
(2) Work done by applying variable force $F(x)$ is $W = \int_{x_{\text{initial}}}^{x_{\text{final}}} F(x)\,\mathrm{d}x$.

Today: Force will be constant (gravity), body will be compound, parts will move different distances.

Math 101 – WORKSHEET 19
MORE WORK

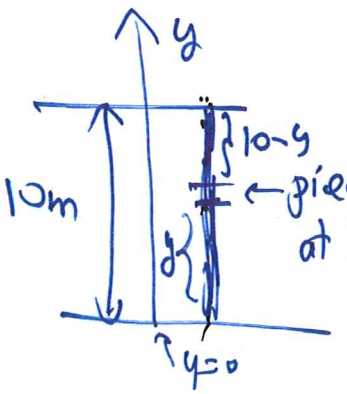
(1) (Preliminary) A worker carries a 20kg bucket to the top of a 10m tall building. Half way up the worker picks up a second 20kg bucket. Calculate the total work done by the worker by adding the contributions from carrying each bucket separately.

first bucket: $20\text{kg} \cdot 9.8\frac{\text{m}}{\text{sec}^2} \cdot 10\text{m} = 1{,}960\text{J}$ (Force, displacement)

second bucket: half that $= 980\text{J}$

total: 2,940J.

(2) (Quiz, 2015) A10m-long cable of mass 7kg is used to lift a bucket off the ground. How much work is needed to raise the entire cable to the height of 10m? Ignore the weight of the bucket, and use $g = 9.8\text{m}/\text{sec}^2$ for the acceleration due to gravity.



The piece of cable at height y, length dy has mass:

$dm = 7\text{kg} \cdot \frac{dy}{10\text{m}}$

(or: density is $\frac{7\text{kg}}{10\text{m}}$ so mass is $\frac{7}{10}dy$)

Date: 17/2/2017, Worksheet by Lior Silberman. This instructional material is excluded from the terms of UBC Policy 81.

Lifting the piece of cable requires work:

$$dW = \underbrace{\frac{7}{10} g \cdot dy}_{\text{force}} \cdot \underbrace{(10-y)}_{\text{distance}}$$

Total work: $\int_{y=0}^{y=10} \frac{7}{10} 9{,}8\,(10-y)\,dy = \frac{7 \cdot 9{,}8}{10} \int_0^{10} (10-y)\,dy =$

$$= \frac{7 \cdot 9{,}8}{10} \left[10y - \frac{1}{2}y^2\right]_0^{10} = 7{,}0 \cdot 5 \cdot 9{,}8 = 343\ \text{J}.$$

Summary:

1. Drew picture
2. Names to quantities (here y distance)
3. Relations between quantities
4. Create integral, calculate integral
5. Solution

(3) (Final, 2012) A tank in the shape of a hemispherical bowl of radius $R = 3\text{m}$ is full of water. It is to be emptied through an outlet extending $H = 2\text{m}$ above its top. Using the values $\rho = 1000\text{kg/m}^3$ for the density of water and $g = 9.8\text{m/s}^2$ for the acceleration due to gravity, find the work (in Joules) required to empty the tank completely. There is no need to simplify your answer but you must evaluate all integrals.