1. Average Value

In this note I collect a few examples of computing the average value of a function, and some example problems using it.

**Definition.** Let \( f \) be defined and integrable on \([a, b] \). The average value of \( f \) on the interval is

\[
\bar{f} = \frac{1}{b-a} \int_a^b f(x) \, dx.
\]

Remark 1. A Riemann sum for \( \int_a^b f(x) \, dx \) is \( \sum_{i=1}^{n} f(x_i^*) \Delta x = \sum_{i=1}^{n} f(x_i^*) \frac{b-a}{n} \); dividing by \( b-a \) we see that a Riemann sum of the integral above is:

\[
\frac{1}{n} \sum_{i=1}^{n} f(x_i^*).\]

In other words, the average value of \( f \) on the interval is the limit of averages of values of \( f \) at sample points.

In straightforward problems you are given \( f, a, b \) and asked to compute the average. In more complicated problems \( a, b \) or \( f \) itself may depend on a parameter, and you need to have the confidence to compute the average in terms of the parameter, getting a formula instead of a numerical answer for the average value. You can then solve for the parameter using given information.
2. Straight-up problems

In these problems, simply compute the average value of the given function on the given interval.

(1) $f(x) = e^{5x} + x\sqrt{x^2 + 1}$ on the interval $[-1, 2]$.

(2) (Final, 2009) $f(\theta) = |\sin \theta - \cos \theta|$ on $[0, \frac{\pi}{2}]$.

(3) (Final, 2011) $f(x) = xe^x$ on $[0, 2]$. 
3. Problems involving a parameter

In the following problems, one piece of information (the function \( f \) or the interval) depends on a parameter. You need to compute the average value using the parameter, and then solve for the parameter.

(1) (Final, 2012) Let \( k \) be a positive constant. Find the average value of \( f(x) = \sin(kx) \) on \([0, \pi/k]\).

(2) Let \( f(x) = x \sqrt{x^2 + r^2} \). For what value of \( r > 0 \) is the average value of \( f \) on \([0, 3]\) equal to \( \frac{1}{2} \)?

(3) (Final, 2010) Find a number \( b > 0 \) such that the function \( f(x) = x - 1 \) has average value 0 on the interval \([0, b]\).