MATH 121 – EXERCISE SET ON INTEGRALS
DUE IN CLASS ON WEDNESDAY, APRIL 4TH

1. STANDARD PROBLEMS

1.1. Calculate the following integrals. You will be primarily graded on the correctness of your results.

(1) \( \int_0^1 (x^3 - 2x + 5) \, dx \)
(2) \( \int x \sqrt{x^2 + a^2} \, dx \)
(3) \( \int \frac{1}{\sqrt{x^2 + a^2}} \cos \left( \sqrt{x^2 + a^2} \right) \, dx \)
(4) \( \int \frac{1}{\sqrt{1-x^4}} \, dx \)
(5) \( \int_0^{\pi/2} e^x \cos x \, dx \)
(6) \( \int e^{\sqrt{x}} \, dx \)
(7) \( \int \frac{x+5}{x^2-2x+3} \, dx \)
(8) \( \int x^3 \log x \, dx \)
(9) \( \int \frac{dt}{(e^t + 1) \log(1+x^2)} \)
(10) \( \int \arctan x \, dx \)

2. HYPERBOLIC TRIG FUNCTIONS

The following substitution technique is superior to trig substitutions for expressions of the form \( \sqrt{x^2 \pm a^2} \).

Let \( \cosh t = \frac{e^t + e^{-t}}{2} \), \( \sinh t = \frac{e^t - e^{-t}}{2} \), \( \tanh x = \frac{\sinh x}{\cosh x} \). The following points are not for submission.

- Verify for yourself that \( \cosh t \geq 1 \) for all \( t \) and that \( \cosh t \) is an even function while \( \sinh t \) is odd.
- Verify that \( (\cosh t)' = \sinh t \) and that \( (\sinh t)' = \cosh t. \)
- Verify the key identity \( \cosh^2 t - \sinh^2 t = 1 \), that is \( \cosh^2 t = 1 + \sinh^2 t \) and \( \sinh^2 t = \cosh^2 t - 1. \)
- Express the equation \( x = \cosh t \) as a quadratic in \( e^t \) and verify that \( t = \pm \log \left( x + \sqrt{x^2 - 1} \right) = \log \left( x \pm \sqrt{x^2 - 1} \right). \)
- Conclude similarly that \( \arcsinh x = \log \left( x + \sqrt{x^2 + 1} \right). \)

Calculate the following integrals. You will be primarily graded on the correctness of your results.

(1) \( \int \sqrt{1+x^2} \, dx \)
(2) \( \int \sqrt{4+x^2} \, dx \)
(3) \( \int x^2 \sqrt{x^2 - a^2} \, dx \)

3. A DIFFICULT CHALLENGE

Evaluate the following integral.

(1) \( \int_0^x \frac{\log x}{1+x^2} \, dx. \)