

Math 100 §105, Fall Term 2010
Midterm Exam

October 4th, 2010

Student number:

LAST name:

First name:

Instructions

- Do not turn this page over until instructed. You will have 45 minutes for the exam.
- You may not use books, notes or electronic devices of any kind.
- Solutions should be written clearly, in complete English sentences, showing all your work.
- If you are using a result from the textbook, the lectures or the problem sets, state it properly.

Signature:

1		/18
2		/8
3		/4
4		/10
Total		/40

1 Short-form answers

Show your work and clearly delineate your final answer. Not all problems are of equal difficulty.

[3] a. Evaluate the following limit (or show it does not exist):

$$\lim_{x \rightarrow \infty} \frac{x^3 - \sin x}{2x^3 + 5x + 1}$$

[3] b. Evaluate the following limit (or show it does not exist):

$$\lim_{x \rightarrow 0} \frac{\tan x}{x}$$

[3] c. Evaluate the following limit (or show it does not exist):

$$\lim_{x \rightarrow 0} \frac{\sqrt{1 + 2x^2} - \sqrt{1 + x^2}}{x^3}$$

[3] d. Differentiate the following function:

$$(1 + x^2 \sin x)^{1/3}$$

[3] e. Differentiate the following function:

$$\frac{e^x + e^{-x}}{2 \cos x}$$

[3] f. Write an equation of the form $y = ax + b$ for the line tangent to the following function at the point $(1, 1)$.

$$y = x^4 - \frac{1}{\pi} \sin(\pi x)$$

2 Long-form answers

A ball falling from rest in air is at height $h(t) = H_0 - gt_0(t + t_0e^{-t/t_0} - t_0)$ at time t . Here H_0 is the initial height, g is the gravitational constant and t_0 depends on the body.

[3] a. Find the velocity $v(t)$ of the ball.

[2] b. Find its acceleration $a(t)$.

[1] c. Find $v(0)$, $a(0)$.

[2] d. Find $\lim_{t \rightarrow \infty} v(t)$.

3 Long-form answers

[4] Let $f(x)$ be a function defined for $0 \leq x \leq 10$. You are given that $f(5) = 1$ and that $f'(5)$ exists and equals 8. Using only the definition of the derivative, evaluate $h'(5)$ where $h(x) = (f(x))^2$.

4 Long-form answers

The function $f(x)$ is defined for non-zero x by

$$f(x) = \begin{cases} ax^2 + bx + c & x < 0 \\ 2 + x^3 \cos(x^{-1}) & x > 0 \end{cases} .$$

[5] a. Determine all values (if any exist) of the constants a, b, c so that $f(x)$ can be made continuous for all x by choosing $f(0)$ appropriately. (Don't forget to justify your answer!)

[5] b. Choosing $f(0)$ as above, determine all values (if any exist) of the constants a, b, c so that $f'(x)$ is continuous for all x . (Don't forget to justify your answer!)