Math 309: Introduction to knot theory
Additional questions for review, collected from the final lectures
(not for submission)

1. Show that every braid diagram \( \beta \) admits and inverse braid diagram \( \beta^{-1} \) so that, after isotopy, \( \beta \cdot \beta^{-1} \) and \( \beta^{-1} \cdot \beta \) (each obtained by concatenation) are equivalent to the trivial braid.

2. Show, via an explicit example, that multiplication in the braid group (on 3 or more strands) is not commutative. For 2-strand braids, multiplication is commutative. Can you explain why?

3. Find the Artin combed form for the braid \( (\sigma_2 \sigma_1^{-1})^3 \).

4. Find all 14 so-called crossingless matchings of 8 points in the boundary of disk (you can think of these as two sets of 4 points on opposite sides of a square). Recall from class that these matchings are a basis for \( TL_4 \).

5. Given a braid \( \beta \in B_n \), write down a formula for the Jones polynomial \( V_\beta(t) \) in terms of the value \( B(\beta) \in TL_n \). (Hint: try the case \( n = 3 \) first)

6. In the case of \( \beta \in B_3 \), determine the effect of an M2 move on \( B(\beta) \), that is, calculate \( B(\sigma_4^\pm 1 \beta) \).