

Solution of special problems

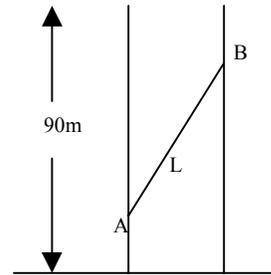
(I) The elevator problem of Quiz II, #5 :

From the hotel lobby on ground floor, Alice takes an elevator to go to the revolving restaurant which is located at 90 meters up above. At the same time, Bob catches an elevator from the restaurant to descend to the lobby. How high is Bob above ground level, at the moment when his distance from Alice is changing at the rate of zero meters per second ? Assume that both elevator rides are smooth, without intermittent stops, and that the elevators move at the same speed.

The Solution : Let  $L$  be the distance between Bob and Alice at time  $t$ . See the attached diagram, which shows 2 elevators moving along two distinct vertical lines. The "moment" corresponds to the instant at which

$\frac{dL}{dt} = 0$ . By critical point theory, that moment is when  $L$  is at a minimum, i.e., when Bob and Alice get closest to each other. Obviously this happens when

they are on the same horizontal plane, each at  $\frac{1}{2} \times 90 = 45$  meters above ground level.



(II) The supersonic flight problem raised as a challenge during class.

The problem : By Air Force regulation, only pilots with special license are allowed to conduct supersonic flights. An urgent mission calls for flying a fighter plane from airbase A to a destination B 2,000 km away. The flight must be accomplished in 1½ hours. Margaret, who doesn't have a special license, applies to conduct this flight. Will her application be granted ?

The Solution : The speed of sound is 331.5 m/s at 0° C. Let us say that in  $t$  seconds Margaret manages to cover a distance of  $f(t)$  meters starting from A. Note that  $1\frac{1}{2} h = 5400s$ . So  $0 \leq t \leq 5400$ . Now  $f(t)$  is a continuous and differentiable function with  $f(0) = 0$ ,  $f(5400) = 2,000,000$ . By the Mean-Value Theorem, there is a time  $t_0$  between 0 and 5400, such that  $\frac{f(5400) - f(0)}{5400 - 0} = f'(t_0) = \text{velocity}$

at time  $t_0$ . But the left hand side is  $\frac{2,000,000 - 0}{5400} = 370.37$  which is greater than the speed of sound ( $= 331.5$  m/s). Thus at time  $t_0$  the flight is truly "supersonic", and Margaret's application will have to be denied.