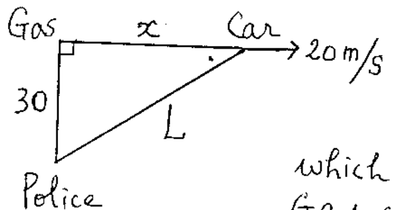
Selected Solutions to Mid-term I

1 (d) The length and width of a rectangle are steadily increasing at the rate of 0.5 cm/min and 0.3 cm/min respectively. At what rate is its area increasing when the rectangle has length 10cm and width 8cm ?

$A = LW$ where L, W are functions of time t.

By product rule $\frac{dA}{dt} = \frac{dL}{dt}W + L\frac{dW}{dt}$, Sub in. $\boxed{(0.5)8 + 10(0.3) = 7}$ cm^2/min

2 (a) A car, travelling east at constant speed of 20 m/s, passes a gas station at time $t = 0$. A policeman standing at 30 meters south of the gas station watches the car with a binocular. At what rate is the distance between the car and the policeman increasing, when the car is exactly 40 meters east of the gas station ?



The diagram shows the situation at a general time instant t, at which the car is, say, x meters east of the Gas station.

Given $x = 20t$ (so $x = 40$ when $t=2$)

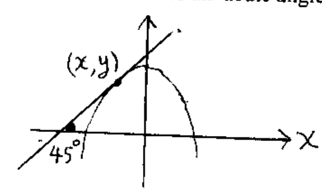
$L = \sqrt{30^2 + x^2} = \sqrt{900 + 400t^2} = (900 + 400t^2)^{\frac{1}{2}}$

$\frac{dL}{dt} = \frac{1}{2}(900 + 400t^2)^{-\frac{1}{2}} \cdot 800t$

Substitute $t = 2$ and simplify to get $\left(\frac{dL}{dt}\right)_{t=2} = 16$ m/s

Ans

2 (b) In the second quadrant, find the point on the parabola $y = \frac{9}{4} - x^2$ where the tangent line makes an acute angle of 45^0 with the positive x - axis. Illustrate with a diagram.



By calculus, slope of tangent line at (x,y) is $y' = -2x$. But this slope is $1 = \tan 45°$. So $-2x = 1$, $x = \frac{-1}{2}$ and $y = \frac{9}{4} - \left(\frac{-1}{2}\right)^2 = 2$. $\boxed{\text{Answer: the point is } \left(\frac{-1}{2}, 2\right)}$

3 (a) Let $f(x) = |\cos x|$. Sketch its graph, and label those points in the interval $[0, 2\pi]$ at which $f(x)$ is not differentiable. No proof is necessary, but the graph is mandatory.